Business Mathematics & Statistics
(MTH 302)

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MTH 302
LECTURE 1
COURSE OVERVIEW

COURSE TITLE
The title of this course is "BUSINESS MATHEMATICS AND STATISTICS".

Instructor's Resume
The instructor of the course is Dr. Zahir Fikri who holds a Ph.D. in Electric Power Systems Engineering from the Royal Institute of Technology, Stockholm, Sweden. The title of Dr. Fikri's thesis was "Statistical Load Forecasting for Distribution Network Planning".

Objective
The purpose of the course is to provide the student with a mathematical basis for personal and business financial decisions through eight instructional modules.
The course stresses business applications using **arithmetic**, **algebra**, **and ratio-proportion** and **graphing**.
Applications include **payroll**, **cost-volume-profit analysis** and **merchandising mathematics**. The course also includes **Statistical Representation of Data, Correlation, Time Series and Exponential Smoothing, Elementary Probability and Probability Distributions.**
This course stresses **logical reasoning** and **problem solving skills**.

Access to **Microsoft Excel** software is required for the course.

Course Outcomes
Successful completion of this course will enable the student to:
1. Apply arithmetic and algebraic skills to everyday business problems.
2. Use ratio, proportion and percent in the solution of business problems.
3. Solve business problems involving commercial discount, markup and markdown.
4. Solve systems of linear equations graphically and algebraically and apply to cost volume-profit analysis.
5. Apply Statistical Representation of Data, Correlation, Time Series and Exponential Smoothing methods in business decision making
6. Use elementary probability theory and knowledge about probability distributions in developing profitable business strategies.

Unit Outcomes Resources/Tests/Assignments
Successful completion of the following units will enable the student to apply mathematical methods to business problems solving.
Required Student Resources (Including textbooks and workbooks)
Text: Selected books on Business Mathematics and Statistics.
Optional Resources
Handouts supplied by the professor.
Instructor's Slides Online or CD based learning materials.
Prerequisites
The students are not required to have any mathematical skills. Basic knowledge of Microsoft Excel will be an advantage but not a requirement.
Evaluation
In order to successfully complete this course, the student is required to meet the following evaluation criteria:
Full participation is expected for this course
All assignments must be completed by the closing date.
Overall grade will be based on VU existing Grading Rules.
All requirements must be met in order to pass the course.

COURSE MODULES

The following are the main modules of this course:

Module 1

- Overview (Lecture 1)
- Perform arithmetic operations in their proper order (Lecture 2)
- Convert fractions their percent and decimal equivalents. (Lecture 2)
- Solve for any one of percent, portion or base, given the other two quantities. (Lecture 2)
- Using Microsoft Excel (Lecture 2)
  Calculate the gross earnings of employees paid a salary, an hourly wage or commissions. (Lecture 3)
- Calculate the simple average or weighted average given a set of values. (Lecture 4)
  Perform basic calculations of the percentages, averages, commission, brokerage and discount (Lecture 5)
- Simple and compound interest (Lecture 6)
- Average due date, interest on drawings and calendar (Lecture 6)

Module 2

- Exponents and radicals (Lecture 7)
- Solve linear equations in one variable (Lecture 7)
- Rearrange formulas to solve for any of its contained variables (Lecture 7)
- Solve problems involving a series of compounding percent changes (Lecture 8)
- Calculate returns from investments (Lecture 8)
- Calculate a single percent change equivalent to a series of percent changes (Lecture 8)
- Matrices (Lecture 9)
- Ratios and Proportions (Lecture10)
- Set up and manipulate ratios (Lecture11)
- Allocate an amount on a prorata basis using proportions (Lecture11)
- Assignment Module 1-2

Module 3

- Discounts (Lectures 12)
- Mathematics of Merchandising (Lectures 13-16)

Module 4

- Applications of Linear Equations (Lecture 17-18)
- Break-even Analysis (Lecture 19-22)
- Assignment Module 3-4
- Mid-Term Examination

Module 5

- Statistical data (Lectures 23)
- Measures of central tendency (Lectures 24-25)
- Measures of dispersion and skewness (Lectures 26-27)

Module 6

- Correlation (Lectures 28-29)
- Line Fitting (Lectures 30-31)
- Time Series and Exponential Smoothing (Lectures 31-33)
- Assignment Module 5-6

Module 7

- Factorials (Lecture 34)
- Permutations and Combinations (Lecture 34)
- Elementary Probability (Lectures 35-36)
- Patterns of probability: Binomial, Poisson and Normal Distributions (Lecture 37-40)

Module 8

- Estimating from Samples: Inference (Lectures 41-42)
- Hypothesis testing : Chi-Square Distribution (Lectures 43-44)
- Planning Production Levels: Linear Programming (Lecture 45)

- Assignment Module 7-8
- End-Term Examination

Note: The course modules are subject to change.

MARKING SCHEME

As per VU Rules

DESCRIPTION OF TOPICS

NO.	MAIN TOPIC	LECTURE	TOPICS	RECOMMENDED READING
Module 1	**1.0 Applications of Basic Mathematics** (Lectures 1-6)	1	• **Overviewew (Lecture** 1)	**Reference 1**
Module 1		2	• **Course Overview** • **Arithmetic Operations &** • **Using Microsoft Excel**	**Reference 2, Lecture 2 Tool: Microsoft Excel**
Module 1		3	• **Calculate Gross Earnings** • **Using Microsoft Excel**	**Reference 2, Lecture 3 Tool: Microsoft Excel**
Module 1		4	• **Calculating simple or weighted averages** • **Using Microsoft Excel**	**Reference 2, Lecture 4 Tool: Microsoft Excel Reference 6**
Module 1		5	• **Basic calculations of percentages, averages, commission, brokerage and discount using** • **Microsoft Excel**	**Reference 2, Lecture 5 Reference 3, Ch 3 Tool: Microsoft Excel**
Module 1		6	• **Simple and compound interest** • **Average due date, interest on drawings and calendar**	**Reference 2, Lecture 6 Reference 3, Ch 3 Tool: Microsoft Excel**
Module 2	**2.0 Applications of Basic Algebra (Lectures 7-9)**	7	• **Exponents and radicals** • **Simplify algebraic expressions** • **Solve linear equations in one variable** • **Rearrange formulas to solve for any of its contained variables**	**Reference 2, Lecture 7 Reference 3, Ch 2 Tool: Microsoft Excel**

		8	• Calculate returns from investments • Problems involving a series of compounding percent changes • Single percent change equivalent to a series of percent changes	Reference 2, Lecture 8 Reference 3, Ch 3 Tool: Microsoft Excel
		9	• Matrices	Reference 2, Lecture 9 Reference 3, Ch 4 Tool: Microsoft Excel
Module 2	3.0 Applications of Ratio and Proportion (Lectures 10-11)	10	• Set up and manipulate ratios. • Set up and solve proportions. • Express percent differences using proportions. • Allocate an amount on a prorata basis using proportions.	Reference 2, Lecture 10 Reference 3, Ch 3 Tool: Microsoft Excel
Module 2		11	• Set up and manipulate ratios. • Allocate an amount on a prorata basis using proportions	Reference 2, Lecture 11 Reference 3, Ch 3 Tool: Microsoft Excel
Module 3	4.0 Merchandising and Financial Mathematics (Lectures 12-16)	12	• Calculate the net price of an item after single or multiple trade discounts. • Calculate an equivalent single discount rate given a series of discounts.	Reference 2, Lecture 12 Reference 3, Ch 3 Tool: Microsoft Excel
Module 3		13	• Solve merchandising pricing problems involving markup and markdown.	Reference 2, Lecture 13 Reference 3, Ch 3 Tool: Microsoft Excel
Module 3		14	• Financial Mathematics Part 1	Reference 2, Lecture 14 Reference 3, Ch 3 Reference 5, Ch 16 Tool: Microsoft Excel
Module 3		15	• Financial Mathematics Part 2	Reference 2, Lecture 15

				Reference 3, Ch 3 **Reference 5, Ch 16** **Tool: Microsoft Excel**
Module 3		16	• **Financial Mathematics Part 3**	**Reference 2, Lecture 16** **Reference 3, Ch 3** **Reference 5, Ch 16** **Tool: Microsoft Excel**
Module 4	**5.0 Break-Even Analysis (Lectures 17-22)**	17	• **Graph a linear equation in two variables.**	**Reference 2, Lecture 17** **Reference 3, Ch 3** **Reference 5, Ch 16 & 18** **Tool: Microsoft Excel**
Module 4		18	• **Solve two linear equations with two unknowns**	**Reference 2, Lecture 18** **Reference 3, Ch 2** **Reference 5, Ch 1** **Tool: Microsoft Excel**
Module 4		19	• **Perform linear cost-volume profit and break-even analysis.** • **Using a break-even chart**	**Reference 2, Lecture 19** **Tool: Microsoft Excel**
Module 4		20	• **Perform linear cost-volume profit and break-even analysis.** • **Using the algebraic approach of solving the cost and revenue functions**	**Reference 2, Lecture 20** **Tool: Microsoft Excel**
Module 4		21	• **Perform linear cost-volume profit and break-even analysis.** • **Using the contribution margin approach**	**Reference 2, Lecture 21** **Tool: Microsoft Excel**
Module 4		22	• **Perform linear cost-volume profit and break-even analysis.** • **Using Microsoft Excel** • **Assignment Module 3-4** • **Mid-Term Examination**	**Reference 2, Lecture 22** **Tool: Microsoft Excel**
Module 5	**6. Statistical Representation of Data (Lectures 23-**	23	• **Statistical Data**	**Reference 2, Lecture 23** **Reference 5, Ch 5** **Tool: Microsoft**

	27)			**Excel**
Module 5		24	• **Statistical Representation Measures of Central Tendency Part 1**	**Reference 2, Lecture 24 Reference 4, Ch 3 Reference 5, Ch 6 Tool: Microsoft Excel**
Module 5		25	• **Statistical Representation** • **Measures of Central Tendency Part 2**	**Reference 2, Lecture 25 Reference 4, Ch 3 Reference 5, Ch 6 Tool: Microsoft Excel**
Module 5		26	• **Measures of Dispersion and Skewness Part 1**	**Reference 2, Lecture 26 Reference 4, Ch 4 Reference 5, Ch 6 Tool: Microsoft Excel**
Module 5		27	• **Measures of Dispersion and Skewness Part 2**	**Reference 2, Lecture 27 Reference 4, Ch 4 Reference 5, Ch 6 Tool: Microsoft Excel**
Module 6	**7. Correlation, Time Series and Exponential Smoothing (Lectures 28-33)**	28	• **Correlation Part 1**	**Reference 2, Lecture 28 Reference 5, Ch 13 Tool: Microsoft Excel**
		29	• **Correlation Part 2**	**Reference 2, Lecture 29 Reference 5, Ch 13 Tool: Microsoft Excel**
		30	• **Line Fitting Part 1**	**Reference 2, Lecture 30 Reference 5, Ch 14 Tool: Microsoft Excel**

		31	• **Line Fitting Part 2**	**Reference 2, Lecture 31 Tool: Microsoft Excel**
		32	• **Time Series and** • **Exponential Smoothing Part 1**	**Reference 2, Lecture 32 Reference 5, Ch 15 Tool: Microsoft Excel**
		33	• **Time Series and** • **Exponential Smoothing Part 2** • **Assignment Module 5-6**	**Reference 2, Lecture 33 Reference 5, Ch 15 Tool: Microsoft Excel**
Module 7	**7. Elementary Probability (Lectures 34-38)**	34	• **Factorials** • **Permutations and Combinations**	**Reference 2, Lecture 34 Reference 3, Ch 2 Tool: Microsoft Excel**
Module 7		35	• **Elementary Probability Part 1**	**Reference 2, Lecture 35 Reference 5, Ch 8 Tool: Microsoft Excel**
Module 7		36	• **Elementary Probability Part 2**	**Reference 2, Lecture 36 Reference 5, Ch 8 Tool: Microsoft Excel**
Module 7		37	• **Patterns of probability: Binomial, Poisson and Normal Distributions Part 1**	**Reference 2, Lecture 39 Reference 5, Ch 9 Tool: Microsoft Excel**
Module 7		38	• **Patterns of probability: Binomial, Poisson and Normal Distributions Part 2**	**Reference 2, Lecture 40 Reference 5, Ch 9 Tool: Microsoft Excel**

Module 7		39	• **Patterns of probability: Binomial, Poisson and Normal Distributions Part 3**	**Reference 2, Lecture 41 Reference 5, Ch 9 Tool: Microsoft Excel**
Module 7		40	• **Patterns of probability: Binomial, Poisson and Normal Distributions Part 4**	**Reference 2, Lecture 41 Reference 5, Ch 9 Tool: Microsoft Excel**
Module 8	**8. Probability Distributions (Lectures 39-44) 9. Linear Programming (Lecture 45)**	41	• **Estimating from Samples: Inference Part 1**	**Reference 2, Lecture 42 Reference 5, Ch 10 Tool: Microsoft Excel**
Module 8		42	• **Estimating from Samples: Inference Part 2**	**Reference 2, Lecture 43 Reference 5, Ch 10 Tool: Microsoft Excel**
Module 8		43	• **Hypothesis testing : Chi-Square Distribution Part 1**	**Reference 2, Lecture 44 Reference 5, Ch 11 Tool: Microsoft Excel**
Module 8		44	• **Hypothesis testing : Chi-Square Distribution Part 2**	**Reference 2, Lecture 45 Reference 5, Ch 11 Tool: Microsoft Excel**
Module 8		45	• **Production Planning: Linear Programming** • **Assignment Module 7-8** • **End Term Examination**	**Reference 2, Lecture 45 Reference 5, Ch 18 Tool: Microsoft Excel**

Methodology

There will be 45 lectures each of 50 minutes duration as indicated above. The lectures will be delivered in a mixture of Urdu and English. The lectures will be heavily supported by slide presentations. The slides for a lecture will be made available on the VU website for the course a few days before the actual lecture is televised. This will allow students to carry out preparatory reading before the lecture. The course will be provided its own page on the VU's web site. This will be used to

provide lecture and other supporting material from the course to the students. The page will have a link to a web-based discussion and bulletin board for the students. Teaching assistants will be assigned by VU to provide various forms of assistance such as grading, answering questions posted by students and preparation of slides.

Grading

There will be a term exam and one final examination. There will also be 4 assignments each covering two modules. The final exam will be comprehensive. These will contribute the following percentages to the final grade:

Mid Term Exam	35%
Final	50%
4 Assignments	15%

Text and Reference Material

The course is based on material from different sources. Topics for reading will be indicated on course web site and in professor's handouts, also to be posted on the course web site. A list of reference books will also be posted and updated on the course web site.

The following material will be used by the students as reference:

Reference 1: Course Outline
2: Instructor's Power Point Slides
3: Business Mathematics & Statistics by Prof. Miraj Din Mirza
4: Elements of statistics & Probability by Shahid Jamal
5: Quantitative Approaches in Business studies by Clare Morris
6: Microsoft Excel Help File

Schedule of Lectures

Given above is the tentative schedule of topics to be covered. Minor changes may occur but these will be announced well in advance.

LECTURE 2
Applications of Basic Mathematics
Part 1

OBJECTIVES

The objectives of the lecture are to learn about:

- Different course modules
- Basic Arithmetic Operations
- Starting Microsoft (MS) Excel
- Using MS Excel to carry out arithmetic operations

COURSE MODULES

This course comprises 8 modules as under:

- Modules 1-4: Mathematics
- Modules 5-8: Statistics

Details of modules are given in handout for lecture 01.

BASIC ARITHMETIC OPERATIONS

Five arithmetic operations provide the foundation for all mathematical operations. These are:

- Addition
- Subtraction
- Multiplication
- Division
- Exponents

Example- Addition

12 + 5 = 17

Example- Subtraction

12 - 5 = 7

Example- Multiplication

12 x 5 = 60

Example- Exponent

(4)^2 = 16

(4)^1/2 = 2

(4)^-1/2 = 1/(4)^1/2 = ½ = 0.5

MICROSOFT EXCEL IN BUSINESS MATHEMATICS & STATISTICS

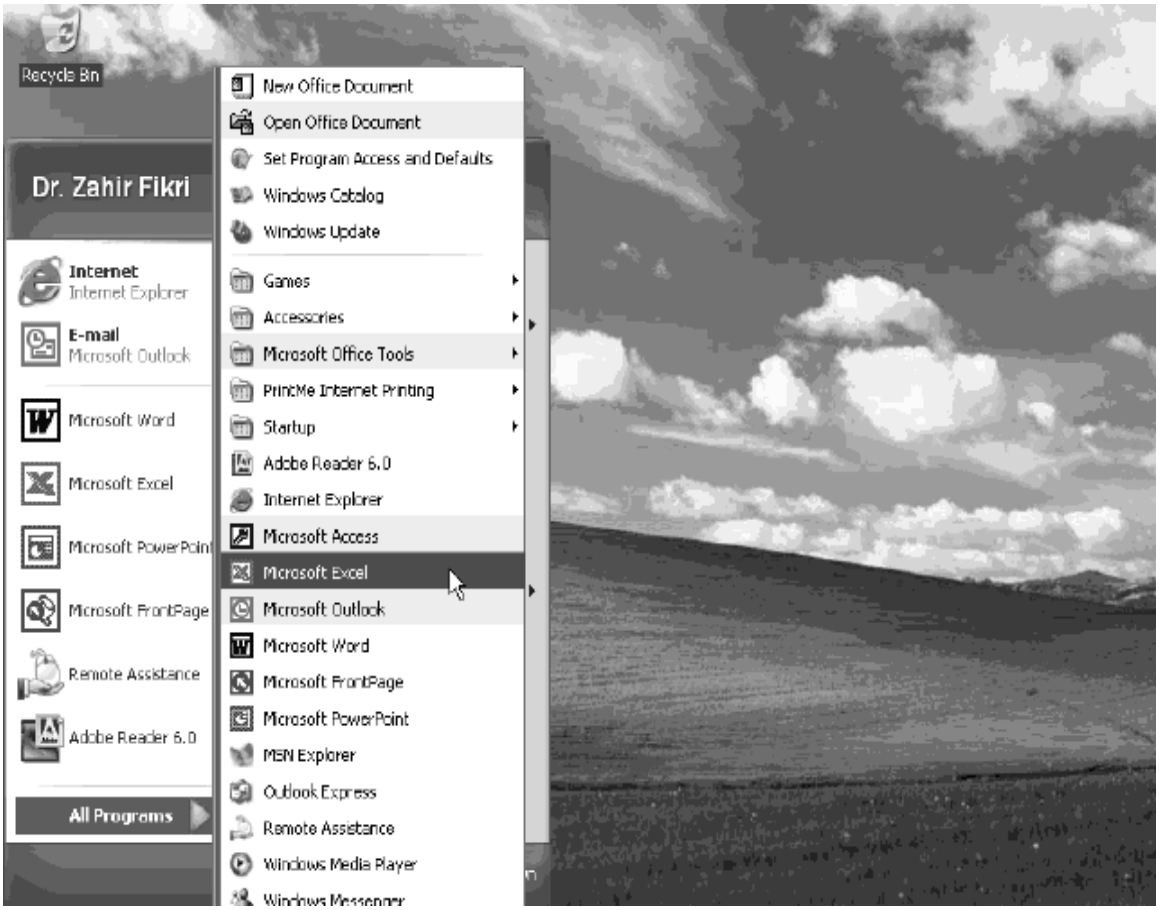
Microsoft Corporation's Spreadsheet software **Excel** is widely used in business mathematics and statistical applications. The latest version of this software is **EXCEL 2002 XP**. This course is based on wide applications of **EXCEL 2002**. It is recommended that you install **EXCEL 2002 XP** software on your computer. If your computer has **Windows 2000** and **EXCEL 2000** even that version of EXCEL can be used as the applications we intend to learn can be done using the earlier version of **EXCEL**. Those of you who are still working with Windows 98 and have **EXCEL 97** installed are encouraged to migrate to newer version of **EXCEL** software.

Starting EXCEL 2000 XP

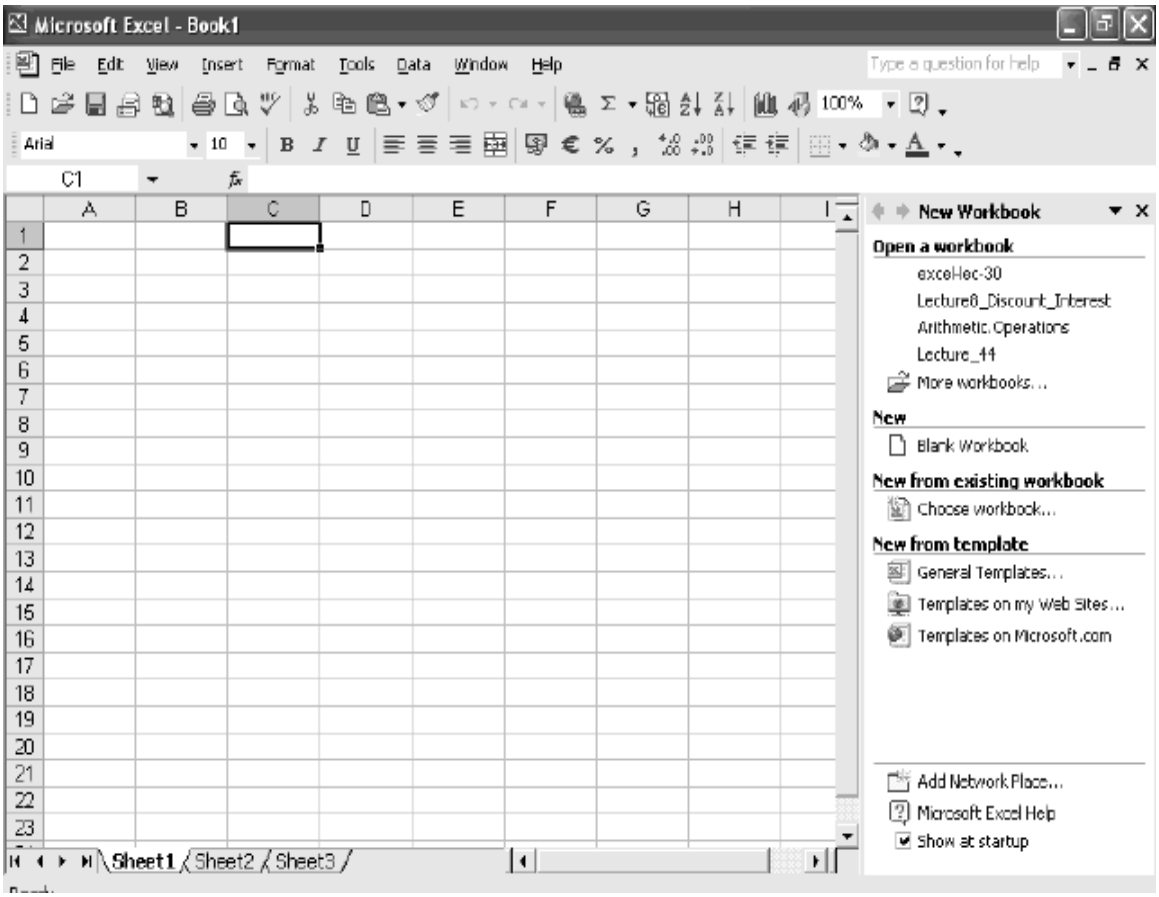
EXCEL 2000 XP can be started by going through the following steps:

1. Click **Start** on your computer
2. Click **All Programs**
3. Click **Microsoft Excel**

The following slides show the operations:



The EXCEL window opens and a blank worksheet becomes available as shown below:



The slide shows a Workbook by the name book1 with three sheets: Sheet1, Sheet2 and Sheet3. The Excel Window has Column numbers starting from A and row numbers starting from 1. the intersection of a row and column is called a **Cell**. The first cell is A1 which is the intersection of column A and row 1. All cells in a Sheet are referenced by a combination of Column name and row number.

Example 1: B15 means cell in column B and row 15.

Example 2: A cell in row 12 and column C has reference C12.

A **Range** defines all cells starting from the leftmost corner where the range starts to the rightmost corner in the last row. The Range is specified by the starting cell, a colon and the ending cell.

Example 3: A **Range** which starts from A1 and ends at D15 is referenced by A1:D15 and has all the cells in columns A to D up to and including row 15.

A value can be entered into a cell by clicking that cell. The mouse pointer which is a rectangle moves to the selected cell. Simply enter the value followed by the **Enter** key. The mouse pointer moves to the cell below.

If you make a mistake while entering the value select the cell again (by clicking it). Enter the new value. The old value is replaced by the new value.

If only one or more digits are to be changed then select the cell. Then double click the mouse. The blinking cursor appears. Either move the arrow key to move to the digit to be

changed or move the cursor to the desired position. Enter the new value and delete the undesired value by using the **Del** key.

I suggest that you learn the basic operations of entering, deleting and changing data in a worksheet.

About calculation operators in Excel

In Excel there are four different types of operators:

1. **Arithmetic operators**
2. **Comparison operators**
3. **Text concatenation operator**
4. **Reference operators**

The following descriptions are reproduced from Excel's Help file for your ready reference. In the present lecture you are directly concerned with arithmetic operators. However, it is important to learn that the comparison operators are used where calculations are made on the basis of comparisons. The text concatenation operator is used to combine two text strings. The reference operators include ":" and "," or ; as the case maybe. We shall learn the use of these operators in different worksheets. You should look through the Excel Help file to see examples of these functions. Selected material from Excel Help File relating to arithmetic operations is given in in a separate file.

The Excel arithmetic operators are as follows:

1. Addition. Symbol: + (Example: =5+4 Result: 9)
2. Subtraction. Symbol: - (Example: =5-4 Result: 1)
3. Multiplication. Symbol: * (Example: =5*4 Result: 20)
4. Division. Symbol: / (Example: =12/4 Result: 3)
5. Percent. Symbol: % (Example: =20% Result: 0.2)
6. Exponentiation: ^ (Example: =5^2 Result: 25)

Excel Formulas for Addition

All calculations in Excel are made through formulas which are written in cells where result is required.

Let us do addition of two numbers 5 and 10.

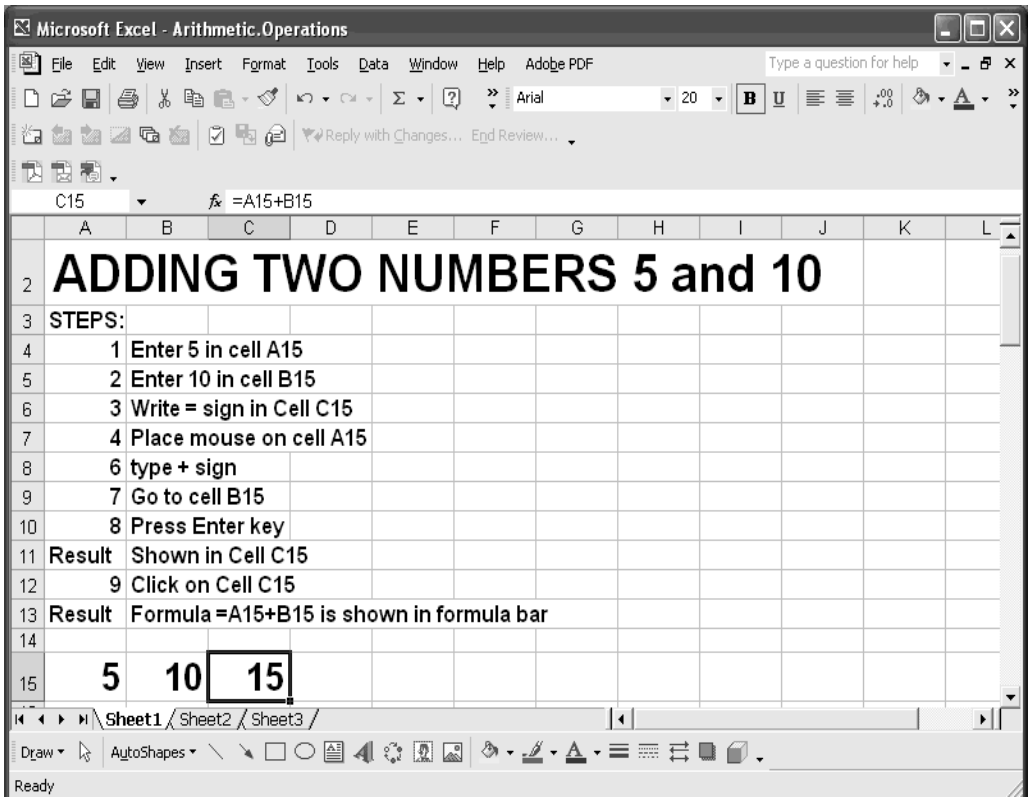
We wish to calculate the addition of two numbers 10 and 5. Let us see how we can add these two numbers in Excel.

1. Open a blank worksheet.
2. Click on a cell where you would like to enter the number 10. Say cell A15.
3. Enter 10 in cell A15.
4. Click cell where you would like to enter the number 5. Say cell B15.
5. Click cell where you would like to get the sum of 10 and 5. Say cell C15.
6. Start the formula. Write equal sign = in cell C15.
7. After =, write "(" (left bracket) in cell C15.
8. Move mouse and left click on value 10 which is in cell A15. In cell C15, the cell reference A15 is written.
9. Write "+" after "A15" in cell C15.
10. Move mouse and left click on value 5 which is in cell B15. In cell C15, the cell reference B15 is written.
11. Write ") " (right bracket) in cell C15.
12. Press Enter key

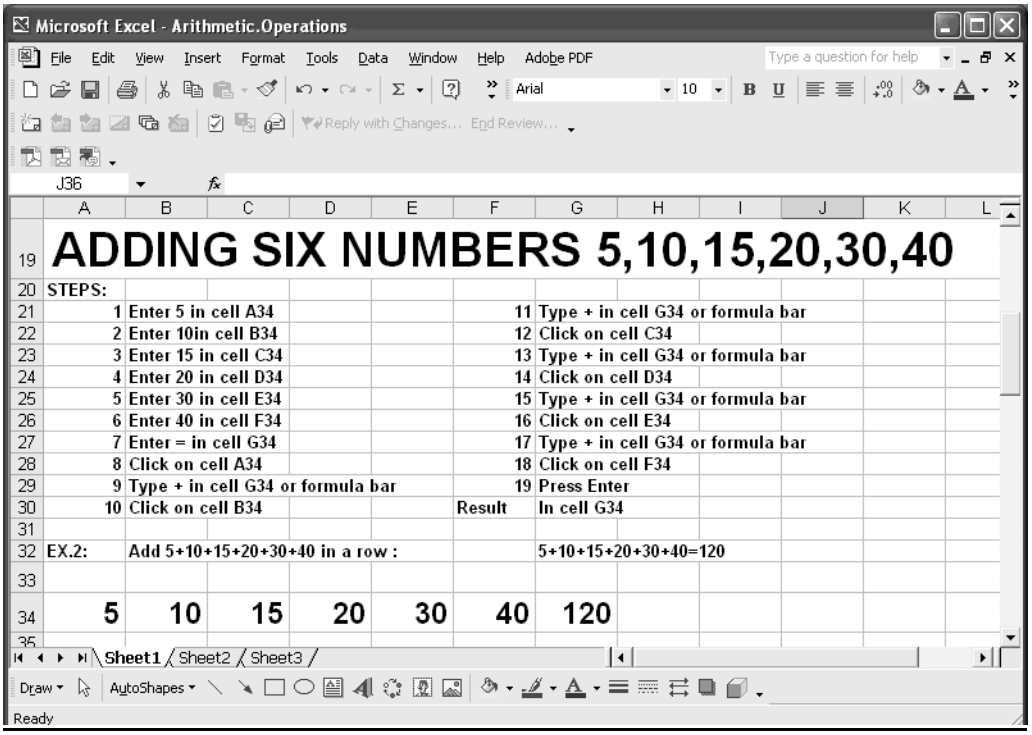
The answer 15 is shown in cell C15.

If you click on cell C15, the formula "=A15+B15" is displayed the formula bar to the right of fx in the Toolbar.

The main steps along with the entries are shown in the slide below. The worksheet MTH302-lec-02 contains the actual entries.



The next slide shows addition of 6 numbers 5, 10, 15, 20, 30 and 40. The entries were made in row 34. The values were entered as follows:
Cell A34: 5
Cell B34: 10
Cell C34: 15
Cell D34: 20
Cell E34: 30
Cell F34: 40
The formula was written in cell G34. The formula was:
=5+10+15+20+30+40
The answer was 120.
You can use an Excel function SUM along with the cell range A34:F34 to calculate the sum of the above numbers. The formula in such a case will be:
=SUM(A34:F34)
You enter "=" followed by SUM, followed by "(". Click on the cell with value 5(reference: A34). Drag the mouse to cell with value 40(reference: F34) and drop the mouse. Enter ")" and then press the Enter key.



In the above two examples you learnt how formulas for addition are written in Excel.

Excel Formula for Subtraction

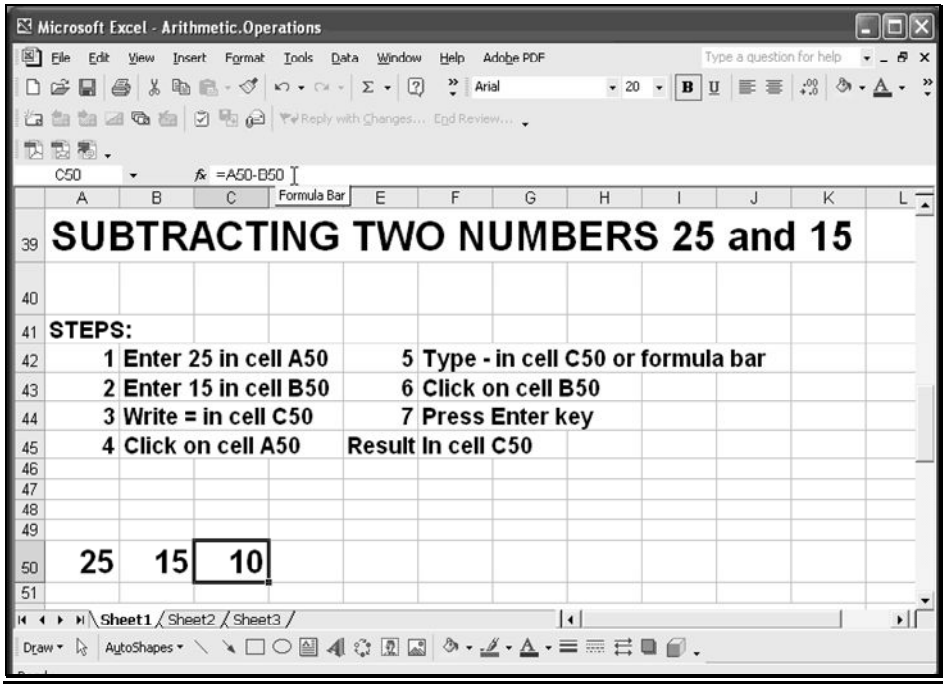
Excel formulas for subtraction are similar to those of addition but with the minus sign. Let us go through the steps for subtracting 15 from 25. Enter values in row 50 as follows:

Cell A50: 25
Cell B50: 15

Write the formula in cell C50 as follows:
=A50-B50

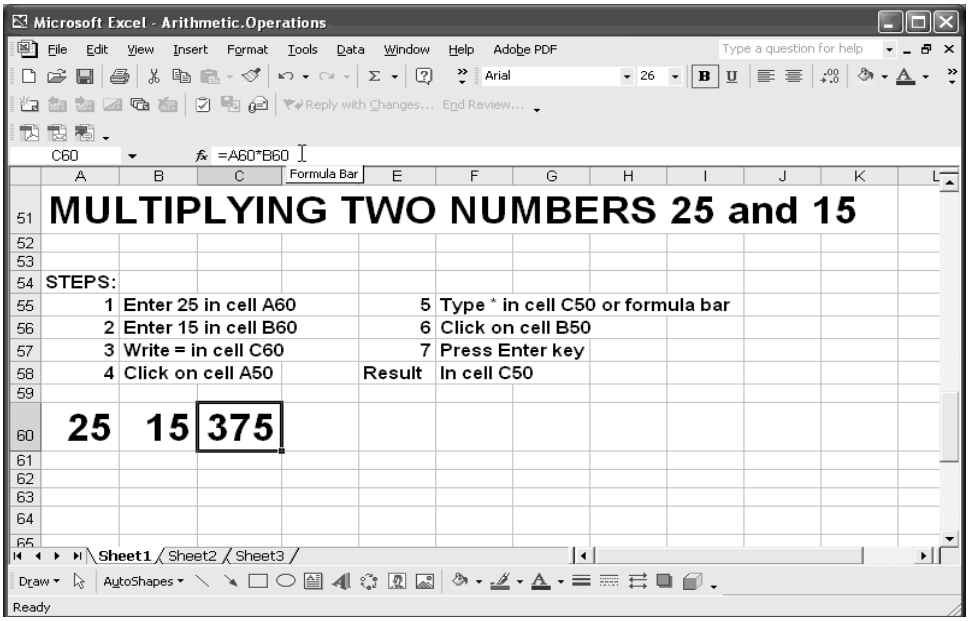
To write this formula, click cell C50, where you want the result. Enter "=". Click on cell with value 25 (reference:A50). Enter "-"(minus sign). Click on cell with value 15 (reference B50). Press enter key.

If you enter 15 first and 25 later, then the question will be to find result of subtraction 15-25.



Excel Formula for Multiplication

Excel formula for multiplication is also similar to the formula for addition. Only the sign of multiplication will be used. The Excel multiplication operator is *.



Let us look at the multiplication of two numbers 25 and 15. The entries will be made in row 60. Enter values as under:

Cell A50: 25

Cell B50: 15

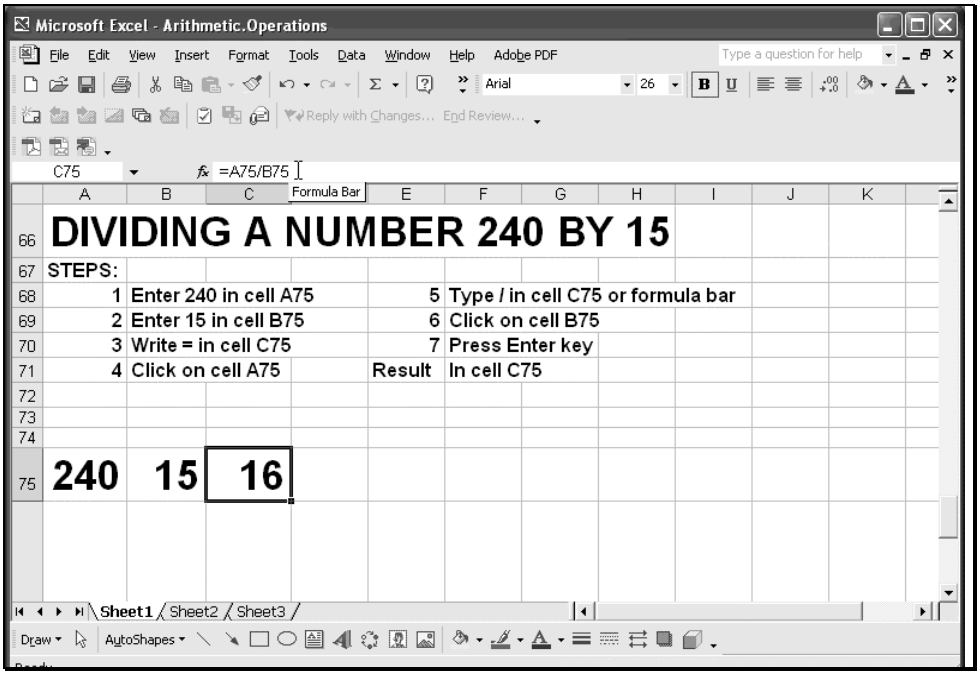
The formula for multiplication is:

=A50*B50

Click on cell C50 to write the formula in that cell. Enter "=". Click on cell with number 25 (reference: A50). Enter "*". Click on cell with number 15 (reference: B50). Press Enter key. The answer is 375 in cell C50.

Excel Formula for Division

The formula for division is similar to that of multiplication with the difference that the division sign "/" will be used.



Let us divide 240 by 15using Excel formula for division. Let us enter numbers in row 75 as follows:

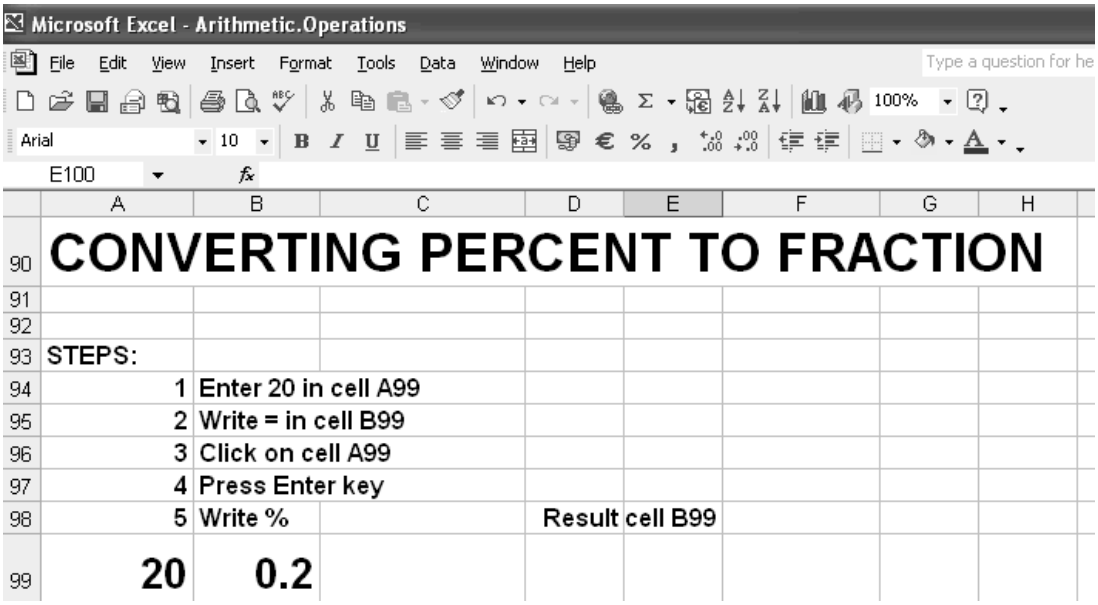
Cell A75: 240
Cell B75: 15

The formula for division will be written in cell C75 as under:
=A75/B75

The steps are as follows: Click the cell A75. Enter 240 in cell A75. Click cell B75. Enter 15. Click cell C75. Enter "=". Click on cell with value 240 (reference: A75). Enter "/". Click cell with number 15 (reference: B75). Press enter key. The answer 16 will be displayed in cell C75.

Excel Formula for Percent

The formula for converting percent to fraction uses the symbol %. To convert 20% to fraction the formula is as under:
=20%
If you enter 20 in cell A99, you can write formula for conversion to fraction by doing the following:
Enter 2o in cell A99. In cell B99 enter "=". Click on cell A99. Enter"%". Press Enter key. The answer 0.2 is given in cell B99.



Excel Formula for Exponentiation

The symbol for exponentiation is ^. The formula for calculating exponents is similar to multiplication with the difference that the carat symbol ^ will be used.

Let us calculate 16 raised to the power 2 by Excel formula for exponentiation. The values will be entered in row 85.

The steps are:

Select Cell A85. Enter 16 in this cell.

Select cell B85 Enter 2 in this cell.

Select cell C85.

Enter"=".

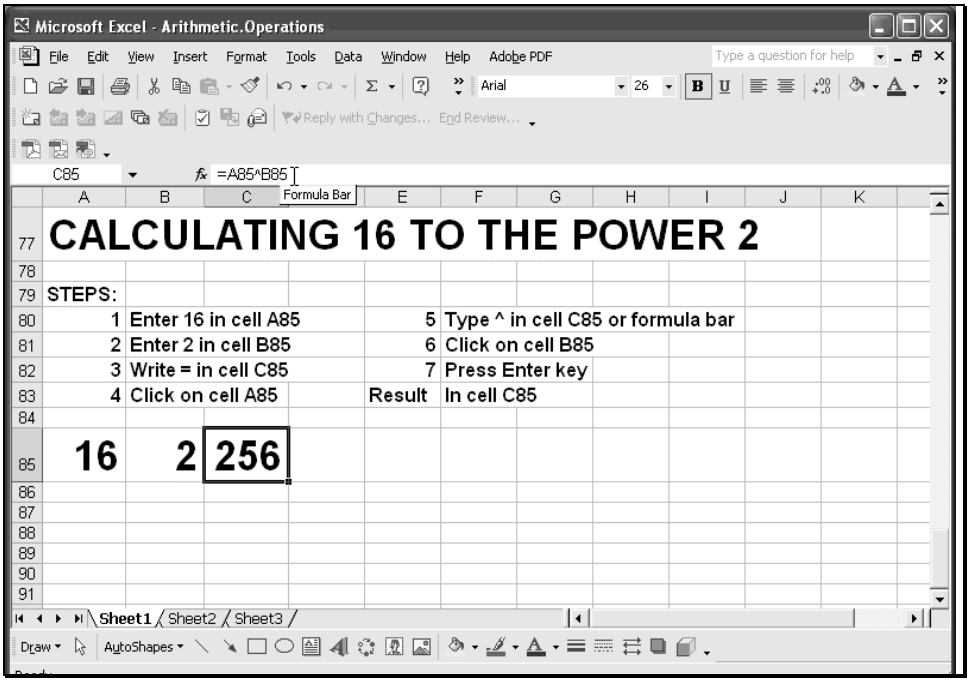
Select cell with value 16 (reference:A85).

Enter "^".

Select number 2 (reference: B85)

Press Enter key.

The result 256 is displayed in cell C85.



Recommended Homework

Download worksheet MTH302-lec-02.xls from the course web site.

- Change values to see change in results.
- Set up new worksheets for each Excel operator with different values.
- Set up worksheets with combinations of operations.

LECTURE 3
Applications of Basic Mathematics
Part 2

OBJECTIVES
The objectives of the lecture are to learn about:
- ***Evaluations***
- ***Calculate Gross Earnings***
- ***Using Microsoft Excel***

Evaluation
In order to successfully complete this course, the student is required to meet the evaluation criteria**:**

- ***Evaluation Criterion 1***
- ***Full participation is expected for this course***

- ***Evaluation Criterion 2***
- ***All assignments must be completed by the closing date***

- ***Evaluation Criterion 3***
- ***Overall grade will be based on VU existing Grading Rules***

- ***Evaluation Criterion 4***
- ***All requirements must be met in order to pass the course***

Grading
There will be a term exam and one final exam; there will also be 4 assignments.
The final exam will be comprehensive.
These will contribute the following percentages to the final grade:
Mid Term Exam 35%
Final 50%
4 Assignments 15%

Collaboration
The students are encouraged to develop collaboration in studying this course. You are advised to carry out discussions with other students on different topics. It will be in your own interest to prepare your own solutions to Assignments. You are advised to make your original original submissions as copying other students' assignments will have negative impact on your studies.

ETHICS
Be advised that as good students your motto should be:
- ***No copying***
- ***No cheating***
- ***No short cuts***

Methodology

There will be 45 lectures each of 50 minutes duration.
The lectures will be delivered in a mixture of Urdu and Englis.

The lectures will be heavily supported by slide presentations.

The slides available on the VU website before the actual lecture is televised.
Students are encouraged to carry out preparatory reading before the lecture.

This course has its own page on the VU's web site.
There are lecture slides as well as other supporting material available on the web site.
Links to a web-based discussion and bulletin board will also been provided.

Teaching assistants will be assigned by VU to provide various forms of assistance such as grading, answering questions posted by students and preparation of slides

Text and Reference Material

This course is based on material from different sources.
Topics for reading will be indicated on course web site and in professor's handouts.
A list of reference books to be posted and updated on course web site. You are encouraged to regularly visit the course web site for latest guidelines for text and reference material.

PROBLEMS

If you have any problems with understanding of the course please contact:

bizmath@vu.edu.pk

GROSS EARNINGS

There may be three types of employees in a company:

- Regular employees drawing a monthly salary
- Part time employees paid on hourly basis
- Payments on per piece basis

To be able to understand how calculations of gross earnings are done, it is important to understand what gross earnings include.
Gross remuneration can include the following:

- Salary
- Provident Fund
- Gratuity Fund
- Social Charges

SALARY

Gross salary includes the following:

- ***Basic salary***
- ***Allowances***
- ***Provident Fund***
- ***Gratuity***
- ***Social Charges***

Gross salary includes:

- **Basic salary**
- **House Rent**
- **Conveyance allowance**
- **Utilities allowance**

Accordance to the taxation rules if allowances are 50% of basic salary, the amount is treated as tax free. Any allowances that exceed this amount, are considered taxable both for the employee as well as the company.

Example 1
The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the taxable income of employee?
Is any add back to the income of the company?
% Allowances = (5000/10000) x 100 =50%
Hence allowances are not taxable.
Total taxable income = 10,000 Rs.
Add back to the income of the company = 0

Example 2
The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 7,000 Rs.
What is the taxable income of employee?
Is any add back to the income of the company?
% Allowances = (7000/10000) x 100 =70%
Allowed non-taxable allowances = 50% = 0.5 x 10000 = 5,000 Rs.
Taxable allowances = 70% – 50% = 7000 - 5000 = 2,000 Rs.
Hence 2000 Rs. of allowances are taxable.
Total taxable income = 10,000 + 2000 = 12,000 Rs.
Add back to the income of the company = 20% allowances = 2,000 Rs.

Structure of Allowances
The common structure of allowances is as under:

- House Rent = 45 %
- Conveyance allowance = 2.5 %
- Utilities allowance = 2.5 %

Example 3
The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the amount of allowances if House Rent = 45 %, Conveyance allowance = 2.5 % and Utilities allowance = 2.5 %?
House rent allowances = 0.45 x 10000 = 4,500 Rs.
Conveyance allowance = 0.025 x 10000 = 250 Rs.
Utilities allowance = 0.025 x 10000 = 250 Rs.

Provident Fund
According to local laws, a company can establish a Provident Trust Fund for the benefit of the employees. By law, $1/11^{th}$ of Basic Salary per month is deducted by the company from the gross earnings of the employee. An equal amount, i.e $1/11^{th}$ of basic salary per month, is contributed by the company to the Provident Fund to the account of the employee. Thus there is an investment of $2/11^{th}$ of basic salary on behalf of the employee in Provident Fund. The company can invest the savings in Provident Fund in Government Approved securities such as defence saving Certificates. Interest earned on investments in Provident Fund is credited to the account of the employees in proportion to their share in the Provident Fund.

Example 4

The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the amount of deduction on account of contribution to the Provident Trust Fund?
What is the contribution of the company?
What is the total saving of the employee per month on account of Provident Trust Fund?
Employee contribution to Provident Fund = 1/11 x 10000 = 909.1 Rs.
Company contribution to Provident Fund = 1/11 x 10000 = 909.1 Rs.
Total savings of employee in Provident Fund = 909.1 + 909.1 = 1,818.2 Rs.

Gratuity Fund

According to local laws, a company can establish a Gratuity Trust Fund for the benefit of the employees. By law, $1/11^{th}$ of Basic Salary per month is contributed by the company to the Gratuity Fund to the account of the employee. Thus there is a saving of $1/11^{th}$ of basic salary on behalf of the employee in Gratuity Fund. The company can invest the savings in Gratuity Fund in Government Approved securities such as defence saving Certificates. Interest earned on investments in Gratuity Fund is credited to the account of the employees in proportion to their share in the Gratuity Fund.

Example 5

The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the contribution of the company on account of gratuity to the Gratuity Trust Fund?
Company contribution to Gratuity Fund
= Total savings of employee in Gratuity Fund = 1/11 x 10000 = 909.1 Rs.

Leaves

All companies have a clear leaves policy. The number of leaves allowed varies from company to company. Typical leaves allowed may be as under:

- ***Casual Leave = 18 Days***
- ***Earned Leave = 18 Days***
- ***Sick Leave = 12 Days***

Example 6

The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the cost on account of casual, earned and sick leaves per year if normal working days per month is 22? What are leaves as percent of gross salary?
Gross salary = 10000 + 5000 = 15,000 Rs.
Casual leaves = (18/22) x 15000 = 12,272.7 Rs.
Earned leaves = (18/22) x 15000 = 12,272.7 Rs
Sick leaves = (12/22) x 15000 = 8,181.8 Rs
Total cost of leaves per year = 12272.7 + 12272.7 + 8181.8 = 32,727.3 Rs.
Total leaves as percent of gross salary = (32727.3/(12 x 15000))x 100 = 18.2%

Social Charges

Social charges comprise leaves, group insurance and medical. Typical medical/group insurance is about 5% of gross salary. Other social benefits may include contribution to

employees children's education, club membership, leave fare assistance etc. Such benefits may be about 5.8%.
Total social charges may therefore may be = 18.2 + 5 + 5.8 = 29%. Other companies may have more social benefits. The 29% social charges are quite common.

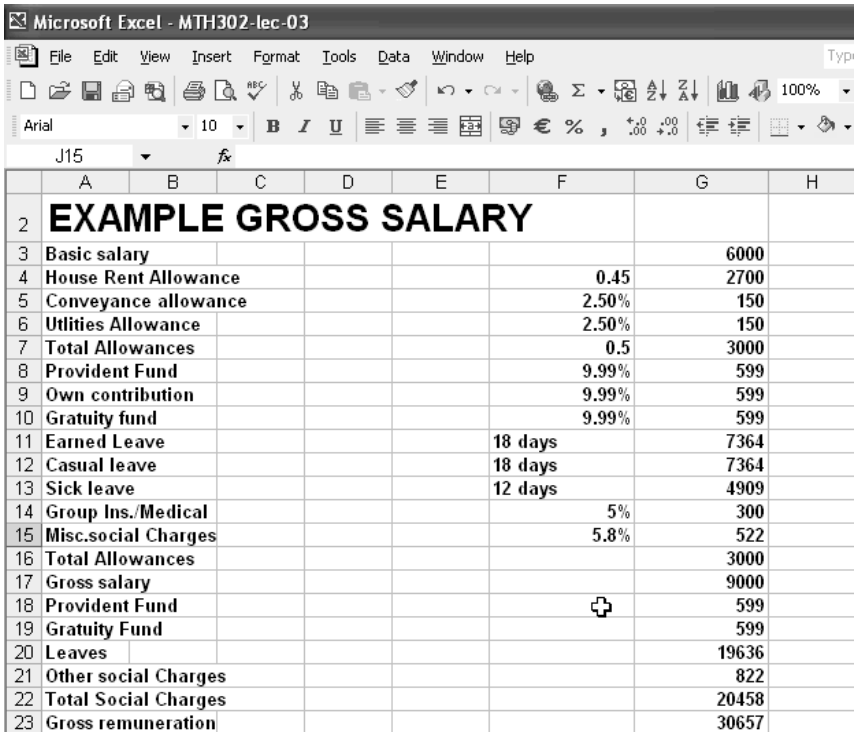
Example 7
The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the cost of the company on account of leaves (18.2%), group insurance/medical (5%) and other social benefits(5.8%)?

Leaves cost = 0.182 x 15000 = 2,730 Rs.
Group insurance/medical = 0.05 x 15000 = 750 Rs.
Other social benefits = 0.058 x 15000 = 870 Rs.
Total social charges = 2730 + 750 + 870 = 4,350 Rs.

Gross Earnings
Summary of different components of salary is as follows:
- ***Basic salary 100 %***
- ***Allowances 50 %***
- ***Gratuity 9.99 %***
- ***Provident Fund 9.99 %***
- ***Social Charges 29 %***

Example 8
The salary of an employee is as follows:
Basic salary = 6,000 Rs.
.

The calculations are shown in the slide below.



	A	F	G
2	EXAMPLE GROSS SALARY		
3	Basic salary		6000
4	House Rent Allowance	0.45	2700
5	Conveyance allowance	2.50%	150
6	Utlities Allowance	2.50%	150
7	Total Allowances	0.5	3000
8	Provident Fund	9.99%	599
9	Own contribution	9.99%	599
10	Gratuity fund	9.99%	599
11	Earned Leave	18 days	7364
12	Casual leave	18 days	7364
13	Sick leave	12 days	4909
14	Group Ins./Medical	5%	300
15	Misc.social Charges	5.8%	522
16	Total Allowances		3000
17	Gross salary		9000
18	Provident Fund		599
19	Gratuity Fund		599
20	Leaves		19636
21	Other social Charges		822
22	Total Social Charges		20458
23	Gross remuneration		30657

Percent from Fraction

Calculate % by multiplying fraction by 100.

Percent = Fraction X 100

Example 9

Convert 0.1 to %.
0.1 X 100= 10%

Common Fraction

Example 10

½ = 0.5
10/100=0.1
Common Fraction
10/100=1/10

Converting % into Common Fraction

Example 11

20%= 20/100= 0.2

Percent

Percent or Fraction Earnings
20% or 20/100=0.2

Base and Rate

Percent of the Base

Example 12

20% of 120?
In 20% of 120:
120 is *Base*
20% is *Rate*

Percentage

Percentage = Base x Rate

Example 13

20% x 120?
20/100 x 120
Or
0.2 X 120
= 24

Example 14

What Percentage is 6 % of 40?

Percentage
= Rate X Base
= 0.06 X 40
= 24

Base

Base = Percentage/Rate

Example 15

Rate = 24.0 %
Percentage = 96
Base= 96/0.24=400

LECTURE 4
Applications of Basic Mathematics

Part 3

OBJECTIVES

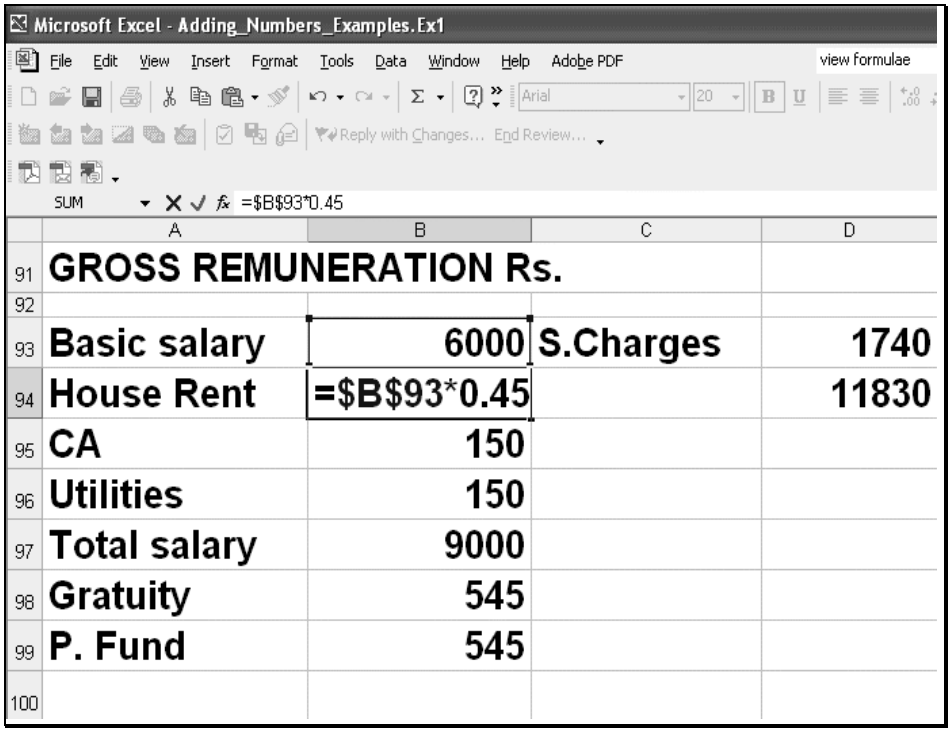
The objectives of the lecture are to learn about:
Review Lecture 3
Calculating simple or weighted averages
Using Microsoft Excel

Gross Remuneration

The following slide shows worksheet calculation of Gross remuneration on the basis of 6000 Rs. Basic salary.

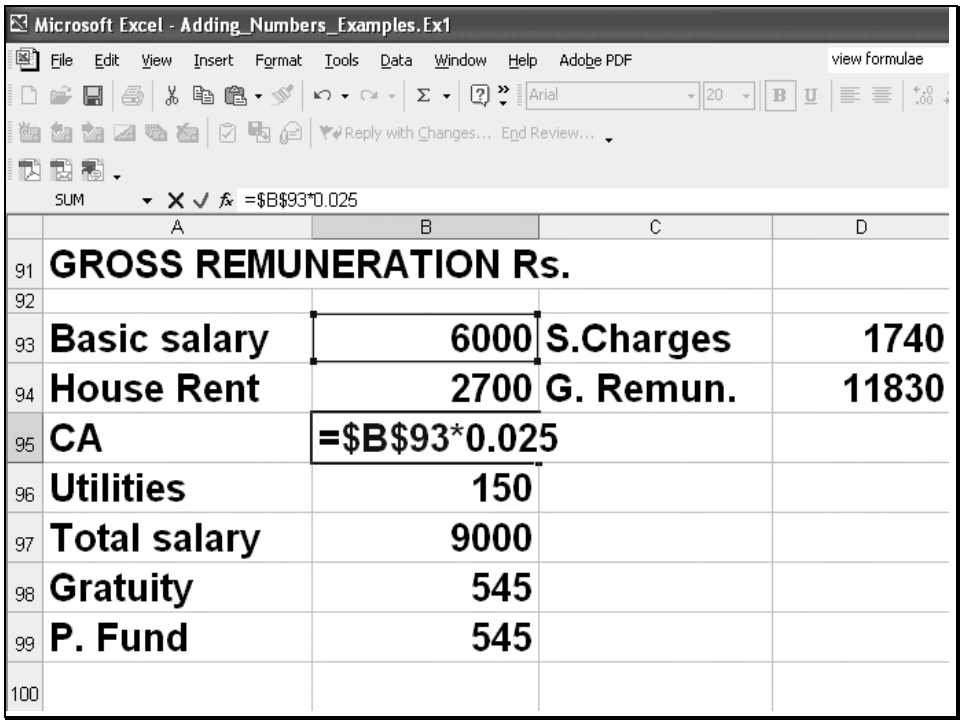
As explained earlier, basic salary is 45% of basic salary. Conveyance and Ultilities Allowance are both 2.5% of basic salary. Both Gratuity and Provident fund are $1/11^{th}$ of basic salary.

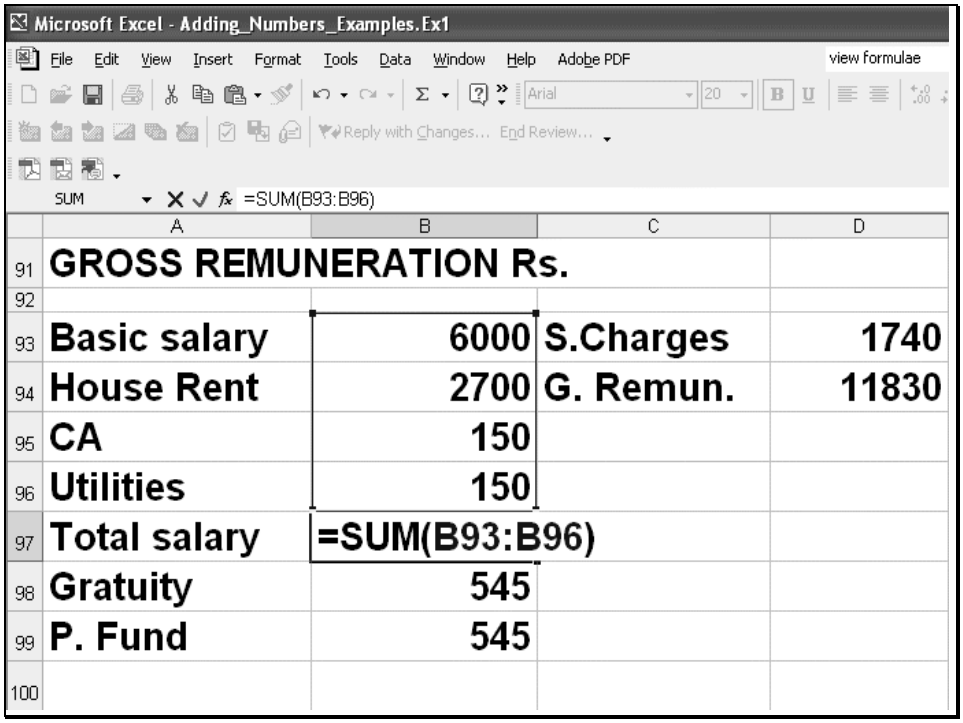
The arithmetic formulas are as follows: Excel forluas are within brackets.
Basic salary = 6000 Rs.
House rent = 0.45 x 6000 = 2700 Rs. (Excel formula: =$B$93*0.45)
Conveyance Allowance = 0.025 x 6000 = 150 Rs. (Excel formula: =$B$93*0.025)
Utilities allowance = 0.025 x 6000 = 150 Rs. (Excel formula: =$B$93*0.025)
Gross salary = 6000 + 2700 + 150 + 150 = 9000 Rs. (Excel formula: =SUM(B93:B96)
Gratuity = 1/11 x 6000 = 545 (Excel formula: =ROUND((1/11)*$B$93;0)



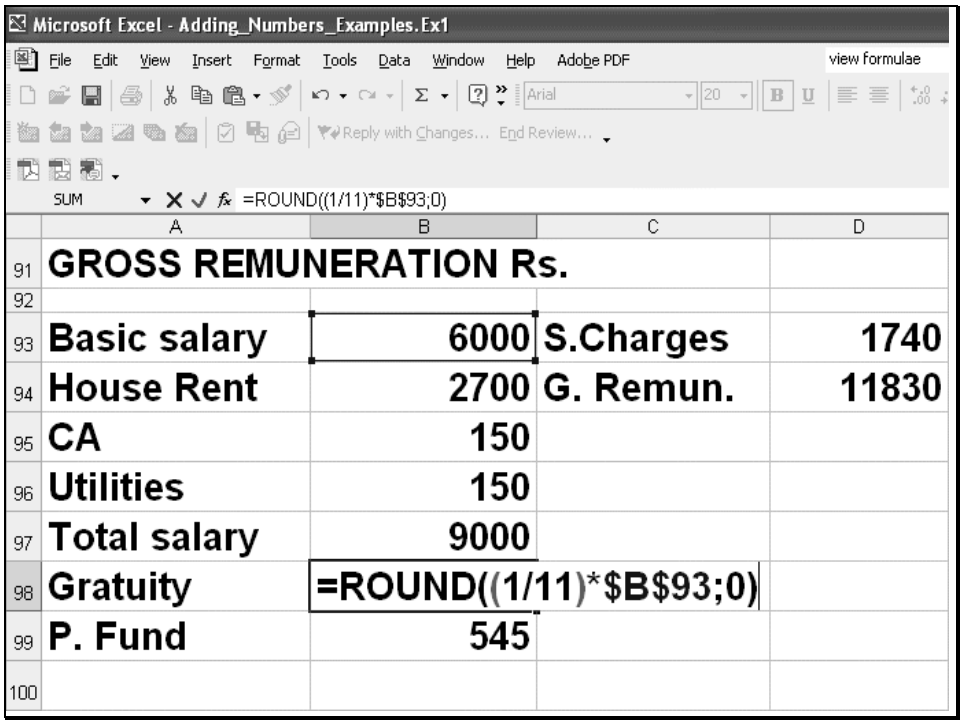
	A	B	C	D
91	GROSS REMUNERATION Rs.			
92				
93	Basic salary	6000	S.Charges	1740
94	House Rent	=$B$93*0.45		11830
95	CA	150		
96	Utilities	150		
97	Total salary	9000		
98	Gratuity	545		
99	P. Fund	545		
100				

In the Excel formulas the $ sign is used before the row and column reference to fix the location of the cell. $B$93 fixes the location of cell B93.

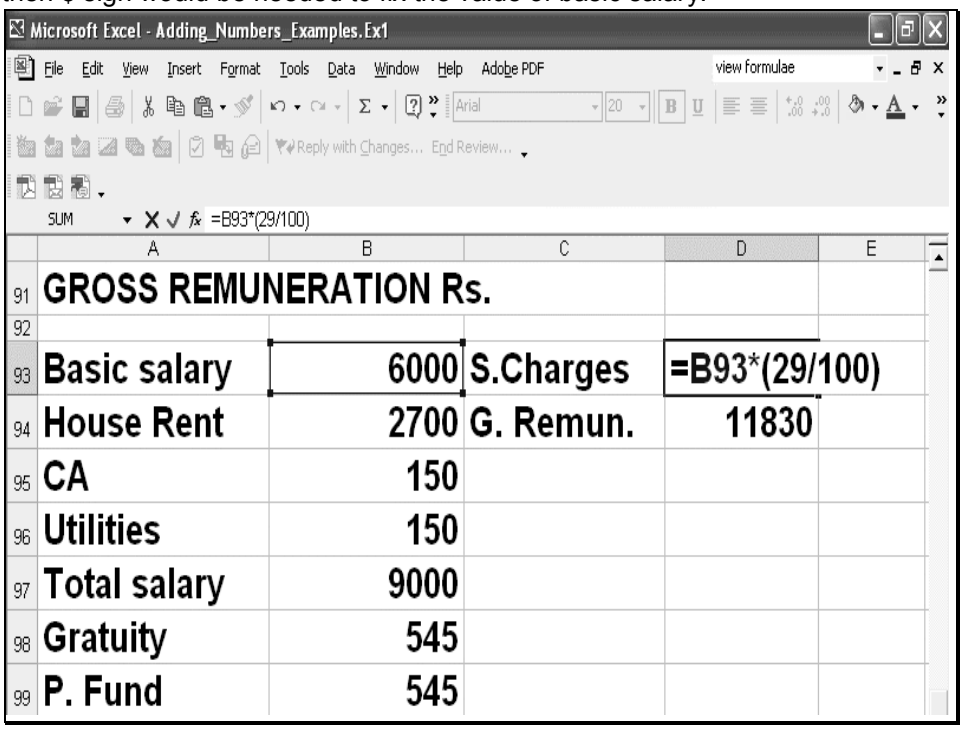


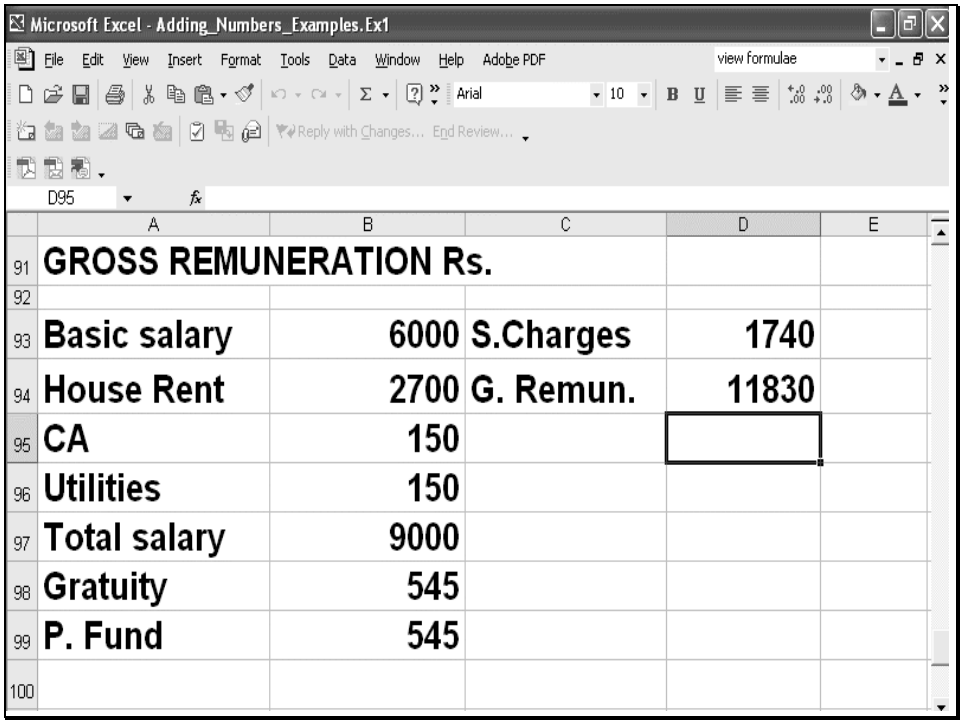


In Gratuity and provident calculations the function ROUND is used to round off values to desired number of decimals. In our case we used the value after the semicolon to indicate that no decimal is required. If you want 1 decimal use the value 1. for 2 decimals use 2 as the second parameter to the ROUND function. The first parameter is the expression for calculation 1/11*$B$93.

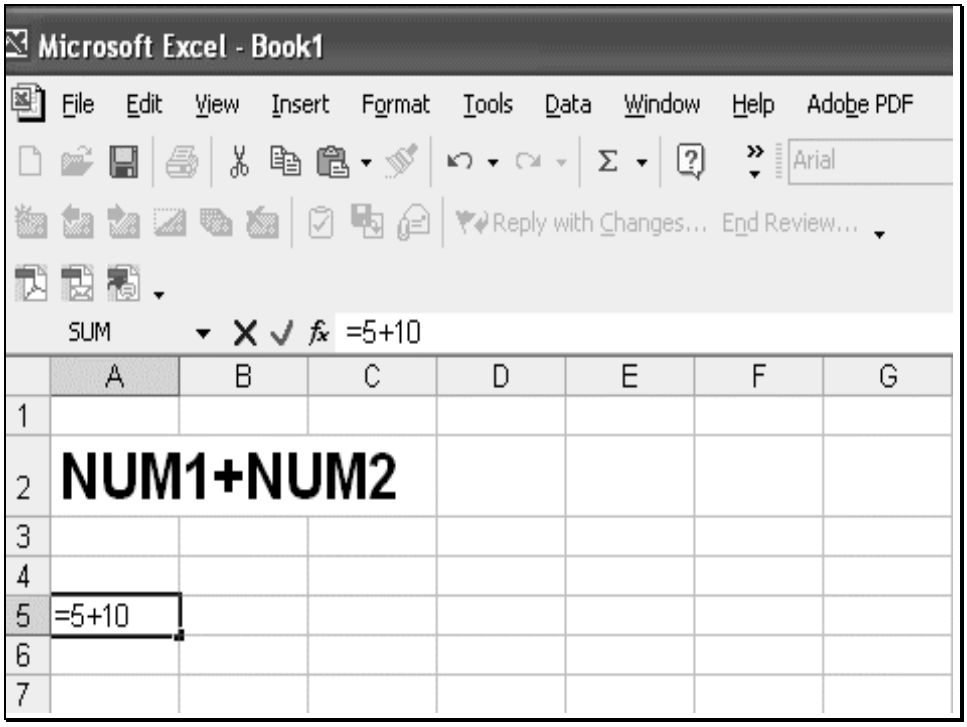


In the calculation for social charges the formula is B93*(29/100). Here 29/100 means 29% social charges. The $ sign was not used here. If the formula was to be copied urther then $ sign would be needed to fix the value of basic salary.

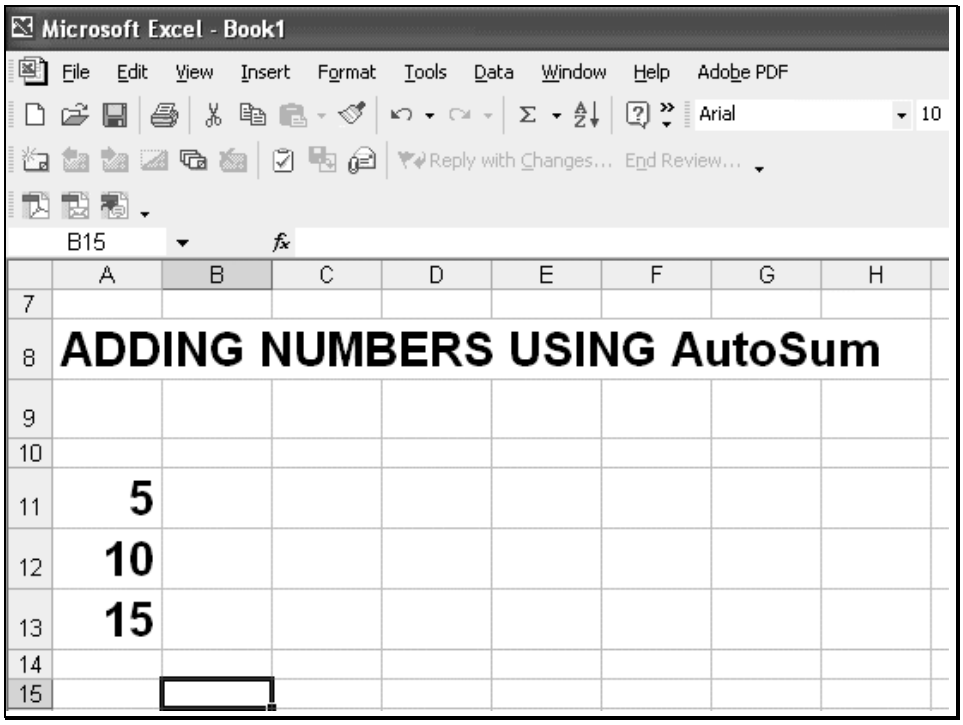




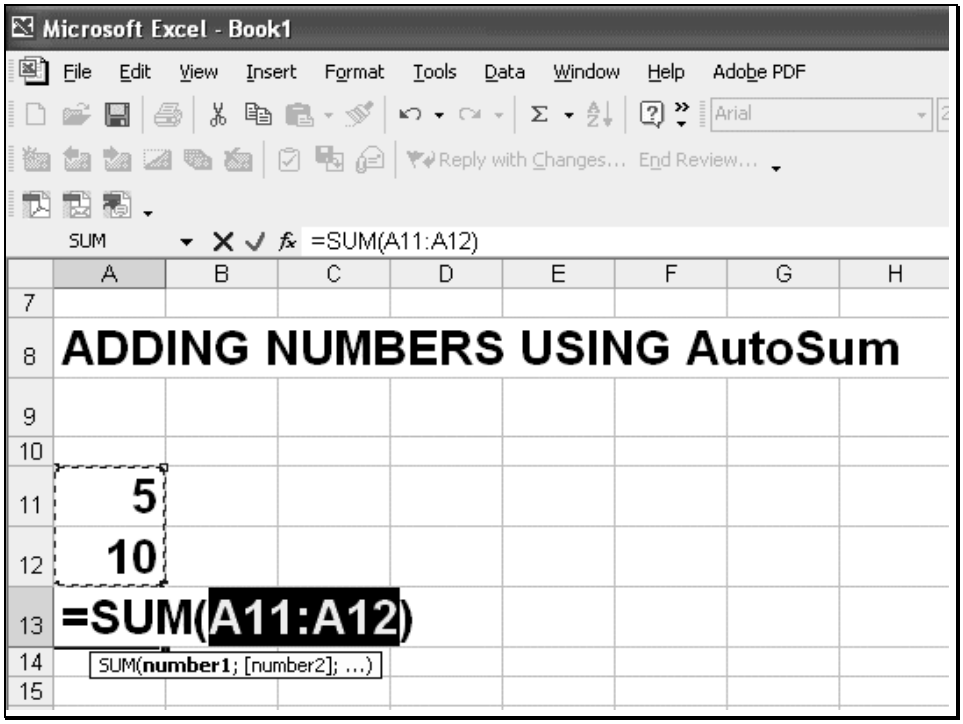
Average
(Arithmetic Mean)
=
Sum /N
Sum= Total of numbers
N= Number of numbers
EXAMPLE 1
Numbers:10, 7, 9, 27, 2
Sum:
= 10+7+9+27+2 = 55
Numbers = 5
Average = 55/5 = 11
ADDING NUMBERS USING MICROSOFT EXCEL
Add numbers as you type them
Add all numbers in a contiguous row or column
Add numbers that are not in a contiguous row or column
Add numbers based on one condition
Add numbers based on multiple conditions
Add numbers based on criteria stored in a separate range
Add numbers based on multiple conditions with the Conditional Sum Wizard
Add numbers
Add numbers as you type them
Type =5+10 in a cell
Result 15.
See Example 2



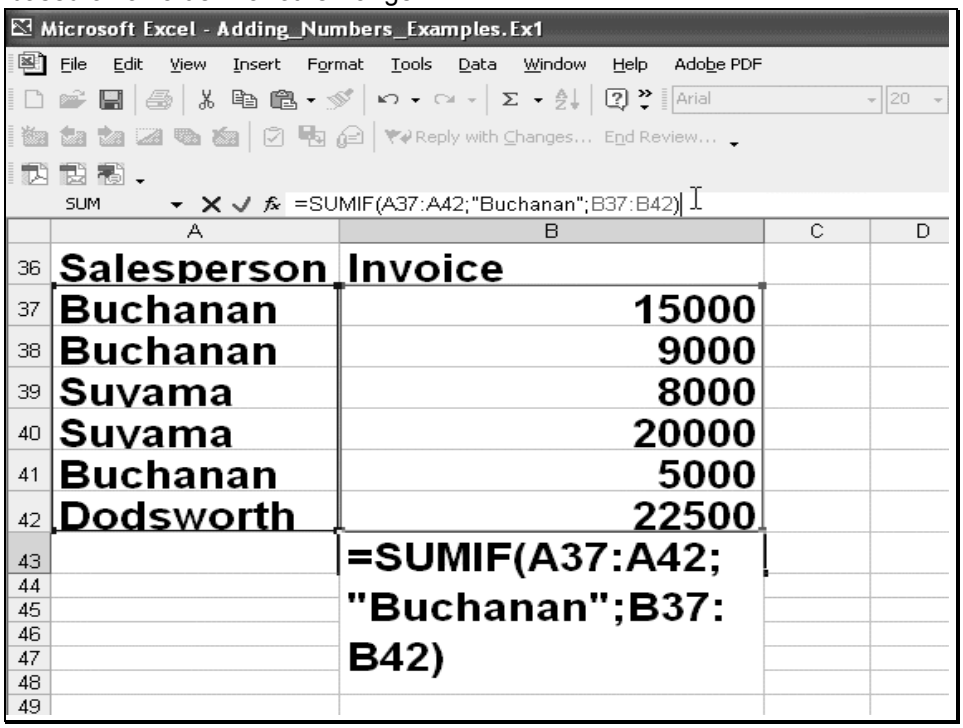
Add all numbers in a contiguous row or column
Click a cell below the column of numbers or to the right of the row of numbers
Click AutoSum
Press ENTER
See Example 2



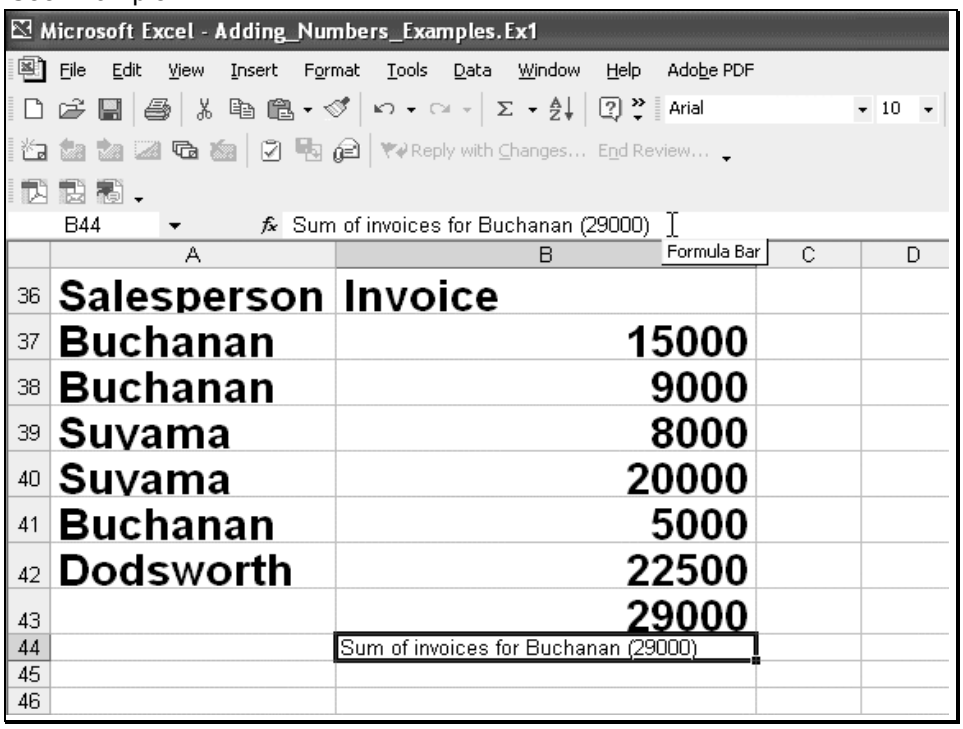
Add numbers that are not in a contiguous row or column
Use the SUM function See Example 3



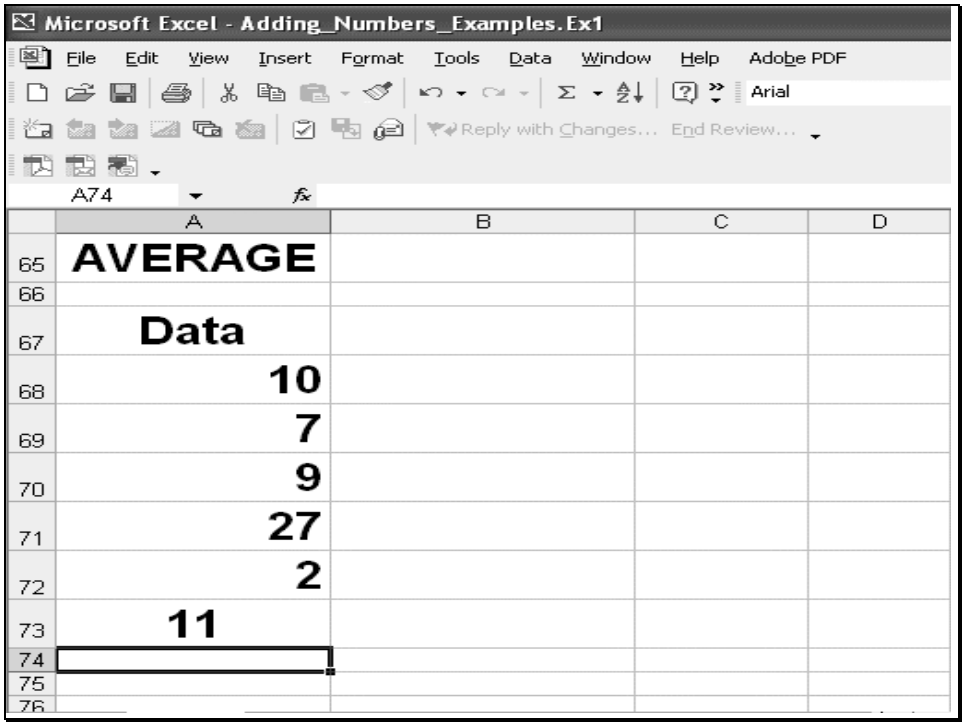
Add numbers based on one condition
Use the SUMIF function
to create a total value
for one range,
based on a value in another range

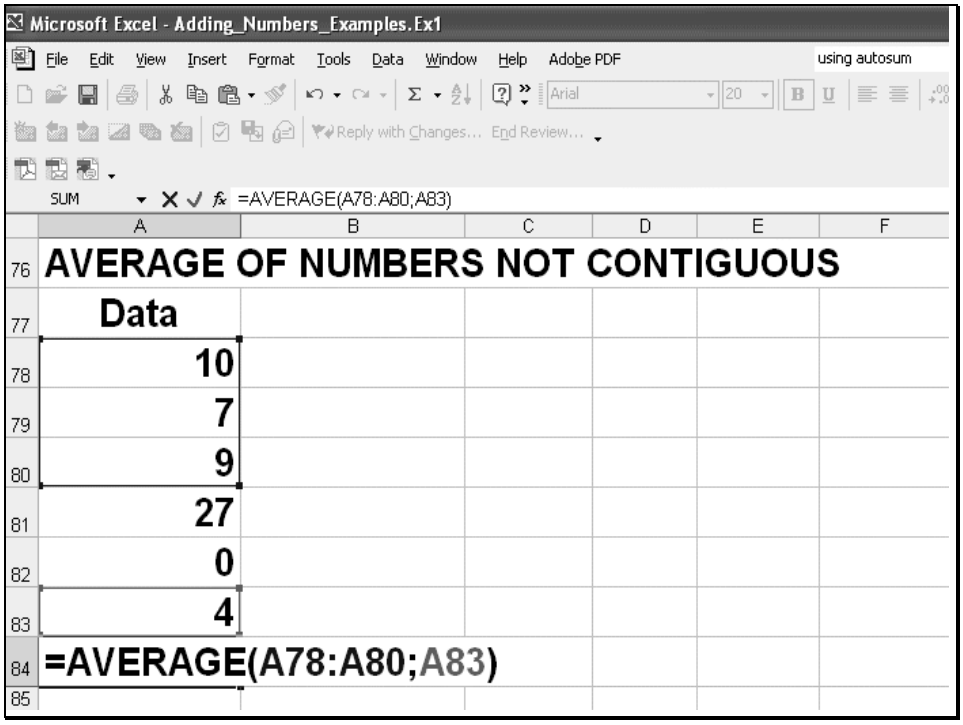


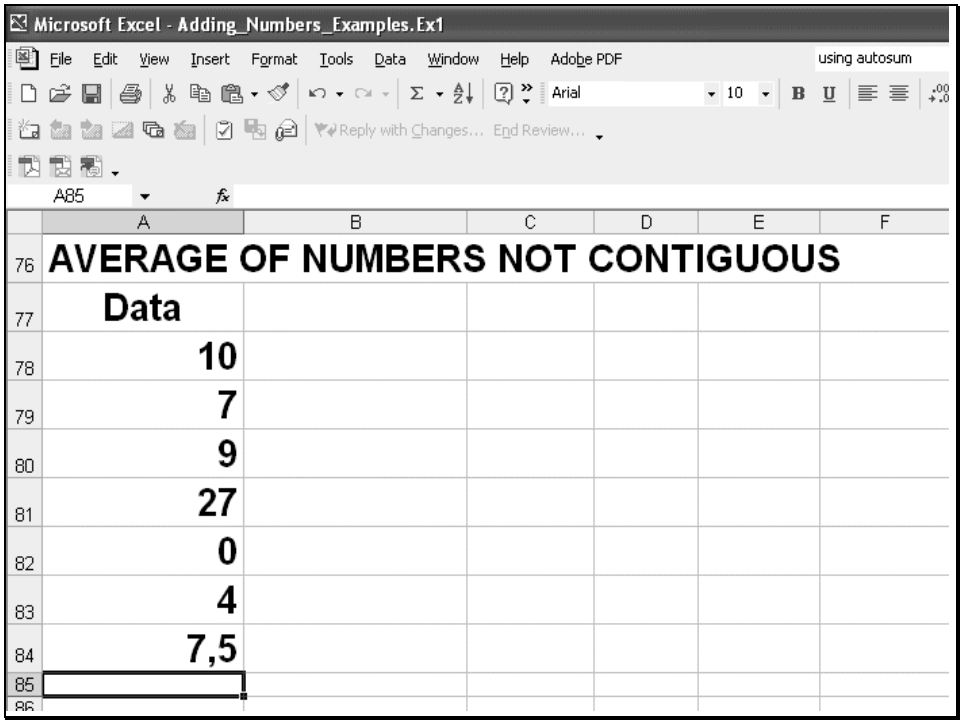
Add numbers based on multiple conditions
Use the IF and SUM functions
to do this task
See Example 4



Add numbers based on criteria stored in a separate range
Use the DSUM function
to do this task
Study DSUM Example
DSUM
Adds the numbers in a column of a list or database that match conditions you specify.
Syntax
DSUM(database,field,criteria)
Database is the range of cells that makes up the list or database.
Field indicates which column is used in the function.
Criteria is the range of cells that contains the conditions you specify.
DSUM
EXAMPLE
=DSUM(A4:E10;"Profit";A1:F2)
The total profit from apple trees with a height between 10 and 16 (75)
AVERAGE USING MICROSOFT EXCEL
Calculate the average of numbers in a contiguous row or column
Calculate the average of numbers not in a contiguous row or column
AVERAGE
Returns the average (arithmetic mean) of the arguments.
Syntax
AVERAGE(number1,number2,...)
Number1, number2, ... are 1 to 30 numeric arguments for which you want the average.







WEIGHTED AVERAGE
Av.1 x weight 1 +
Av. 2 x weight 2 +..
Av. N x weight n
Weights in fractions

LECTURE 5
Applications of Basic Mathematics
Part 4

OBJECTIVES

The objectives of the lecture are to learn about:

- ***Review of Lecture 4***
- ***Basic calculations of percentages, salaries and investments using Microsoft Excel***

PERCENTAGE CHANGE

Monday's Sales were Rs.1000 and grew to Rs. 2500 the next day.
Find the percent change.

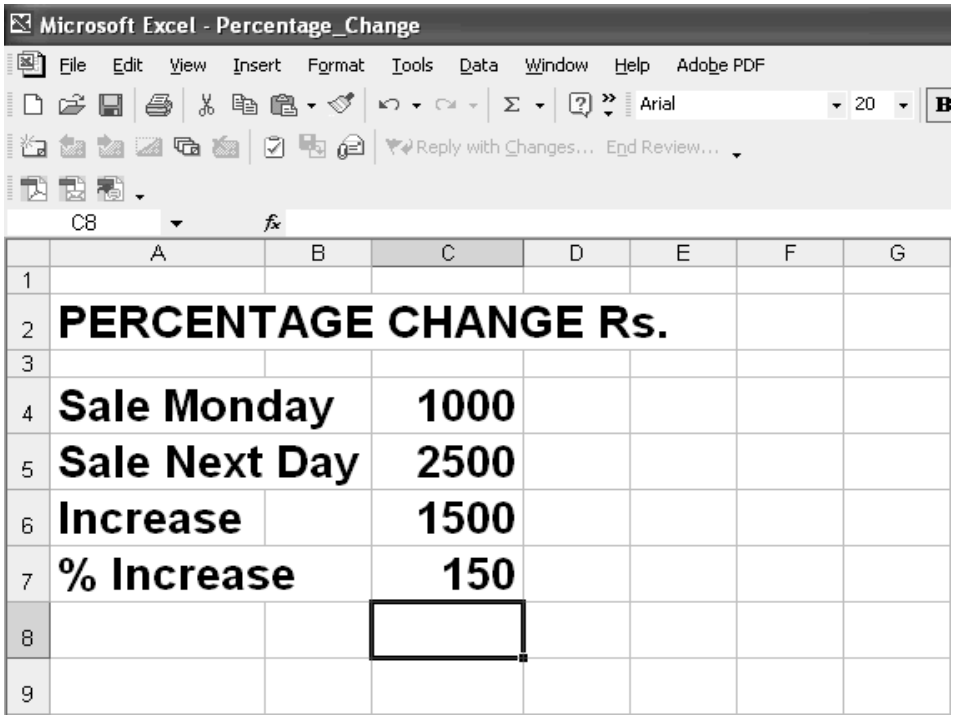
METHOD

Change = Final value – initial value
Percentage change = (Change/initial value) x 100%

CALCULATION

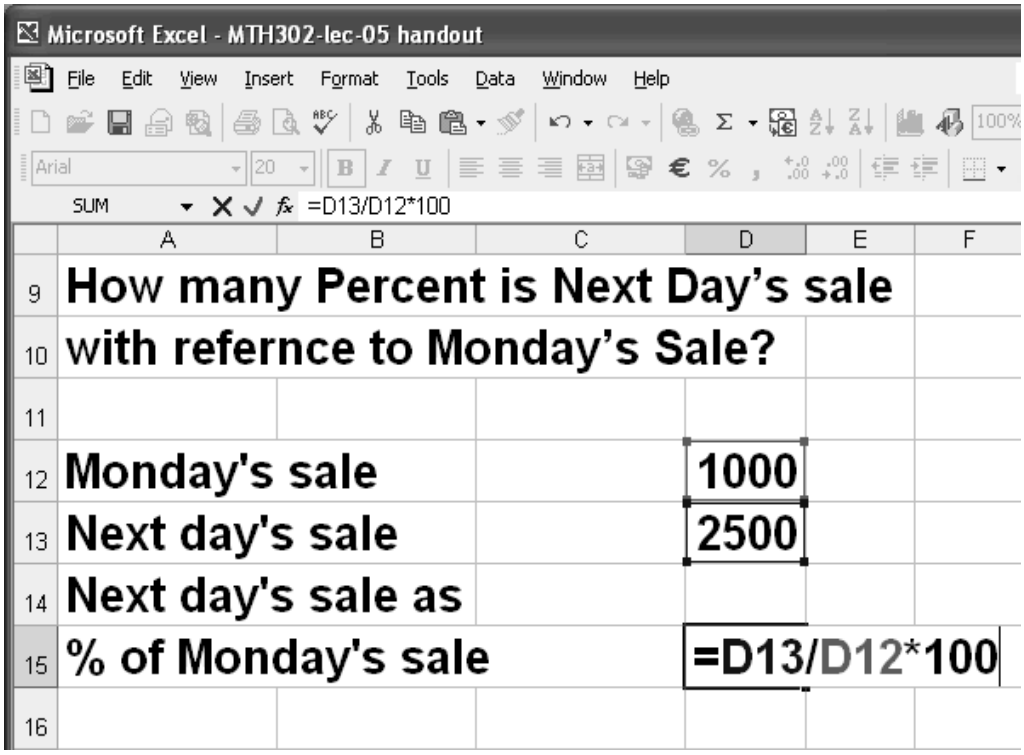
Initial value =1000
Final value = 2500
Change = 1500
% Change = (1500/1000) x 100 = 150%

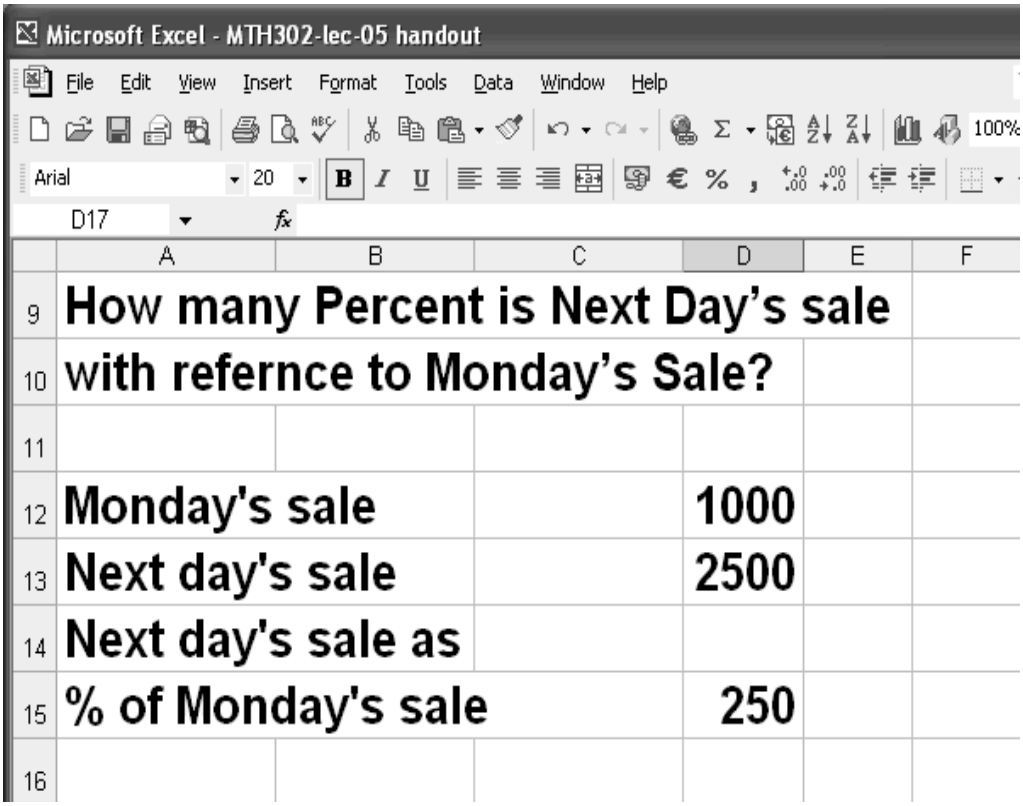
The calculations using Excel are given below.
First the entries of data were made as follows:
Cell C4 = 1000
Cell C5 = 2500
In cell C6 the formula for increase was: =C4-C5
The result was 1500.
In cell C7 the formula for percentage change was: = C6/C4*100
The result 150 is shown in the next slide.



EXAMPLE 1

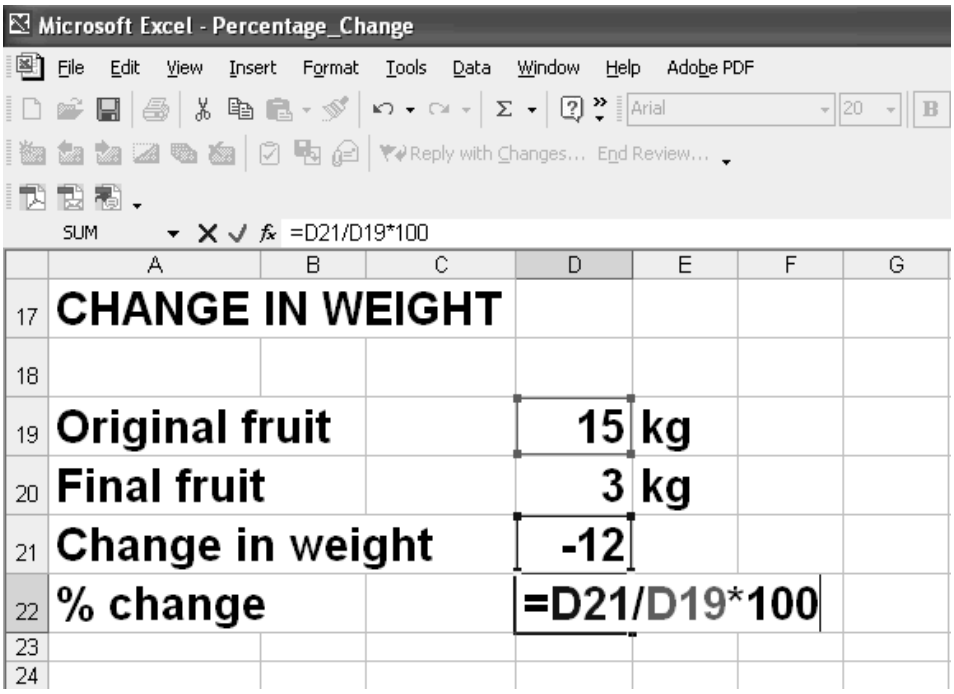
How many Percent is Next Day's sale with reference to Monday's Sale?
Monday's sale= 1000
Next day's sale= 2500
Next day's sale as % = 2500/1000 x 100 = 250 %
= Two and a half times

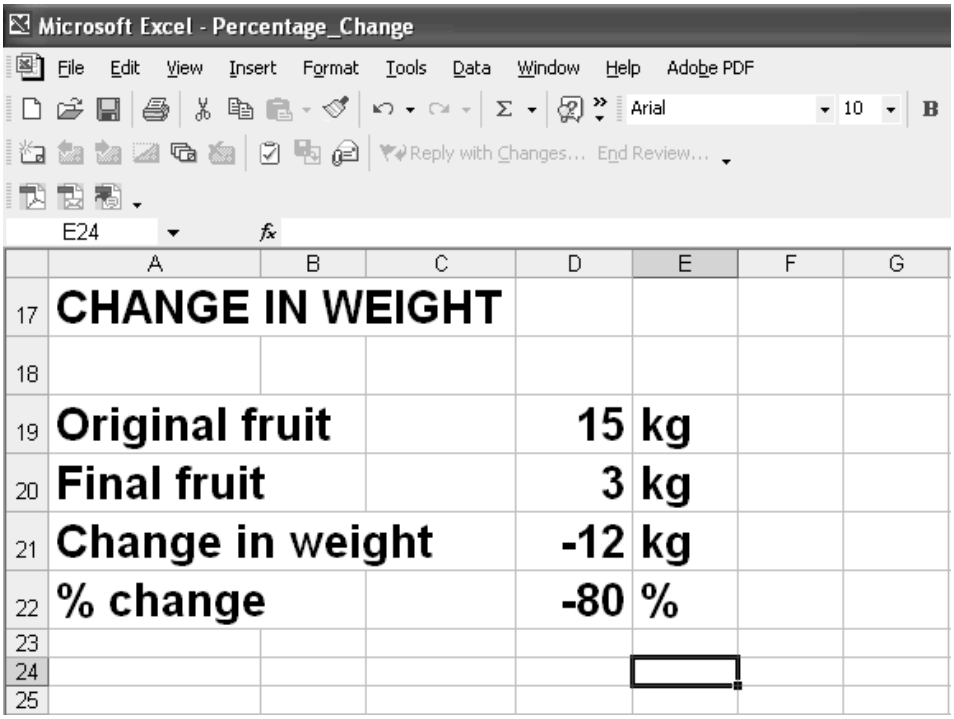




EXAMPLE 2
In the making of dried fruit, 15kg. of fruit shrinks to 3 kg
Find the percent change.

Calculation
Original fruit = 15 kg
Final fruit = 3 kg
Change = 3-15 = -12
% change = - 12/15 x 100 = - 80 %
Size was reduced by 80%





Calculations in Excel were done as follows:

Data entry

Cell D19: 15
Cell D20: 3

Formulas

Formula for change in Cell D21: =D19-D20
Formula for %change in Cell D22: = D21/D19*100

Results

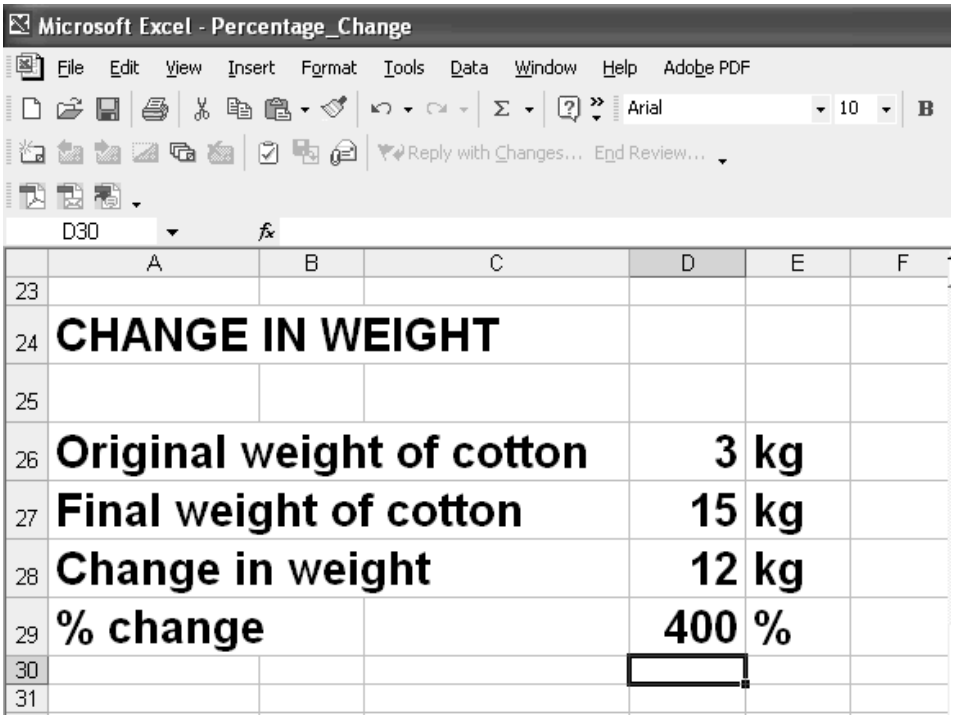
Cell D21 = -12 kg
Cell D22 = -80 %

EXAMPLE 3

After mixing with water the weight of cotton increased from 3 kg to 15 kg. Find the percent change.

CALCULATION

Original weight = 3 kg
Final weight = 15 kg
Change = 15-3= 12
% change = 12/3 x 100 = 400 %
Weight increased by 400%



Calculations in Excel were done as follows:

Data entry
Cell D26: 3
Cell D27: 15

Formulas
Formula for change in Cell D28: =D26-D27
Formula for %change in Cell D29: = D28/D26*100

Results

Cell D28 = 12 kg
Cell D29 = 400 %

EXAMPLE 4

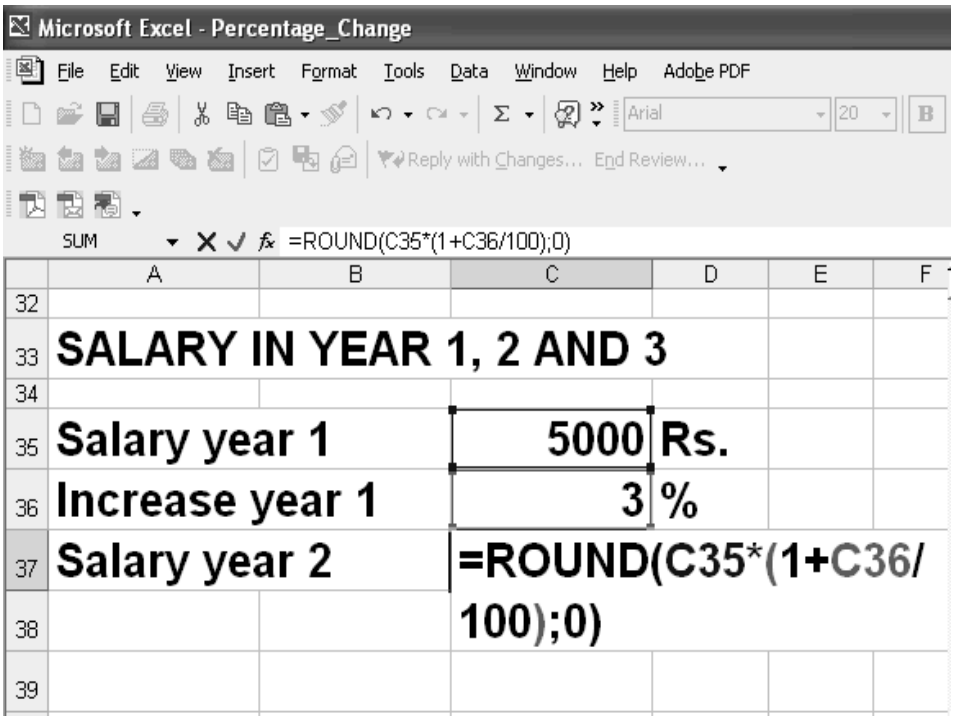
A union signed a three year collective agreement that provided for wage increases of 3%, 2%, and 1% in successive years

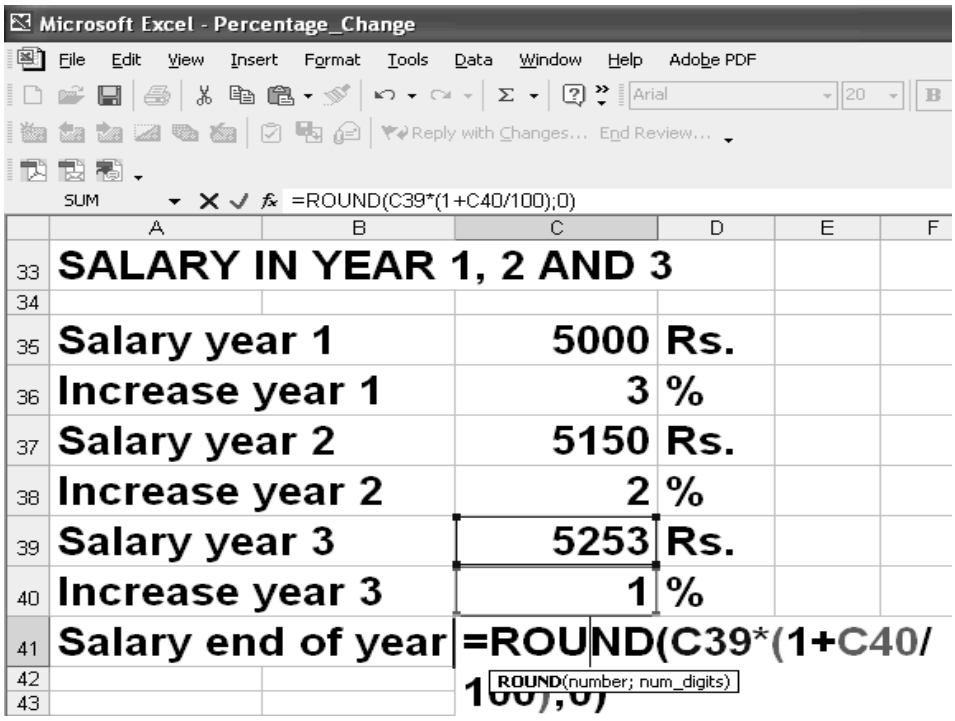
An employee is currently earning 5000 rupees per month
What will be the salary per month at the end of the term of the contract?

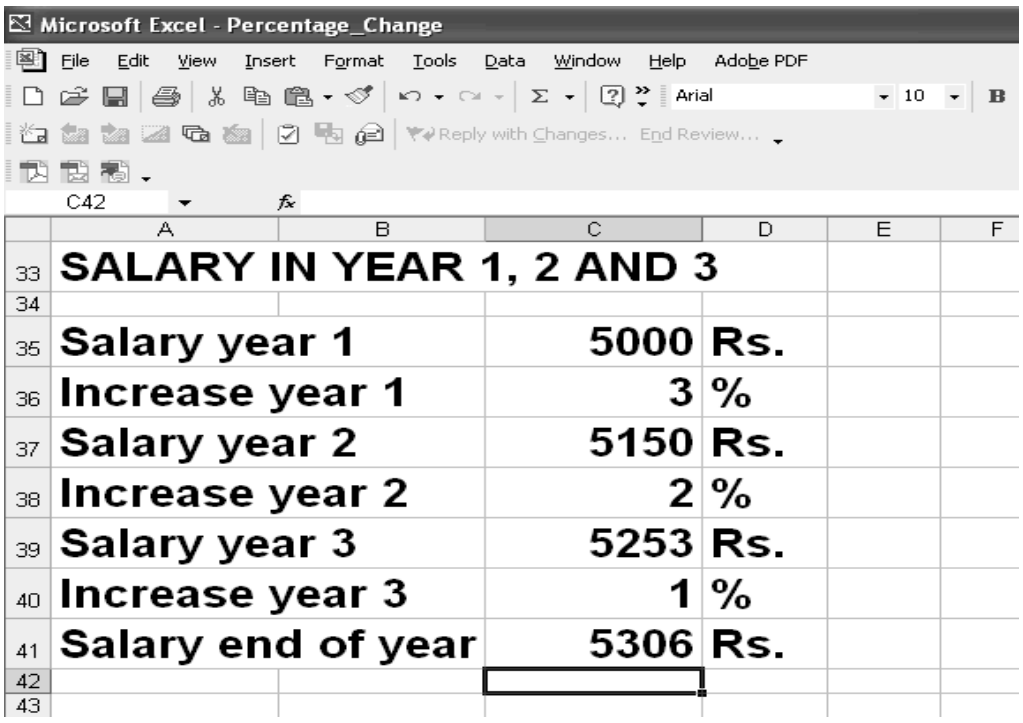
Calculation

= 5000(1 + 3%)(1 + 2%)(1 + 1%)
= 5000 x 1.03 x 1.02 x 1.01
= 5306 Rs.

Calculations using Excel are shown in the following slides.







Calculations in Excel were done as follows:

Data entry

Cell C35: 5000
Cell C36: 3
Cell C38: 2
Cell C40: 1

Formulas

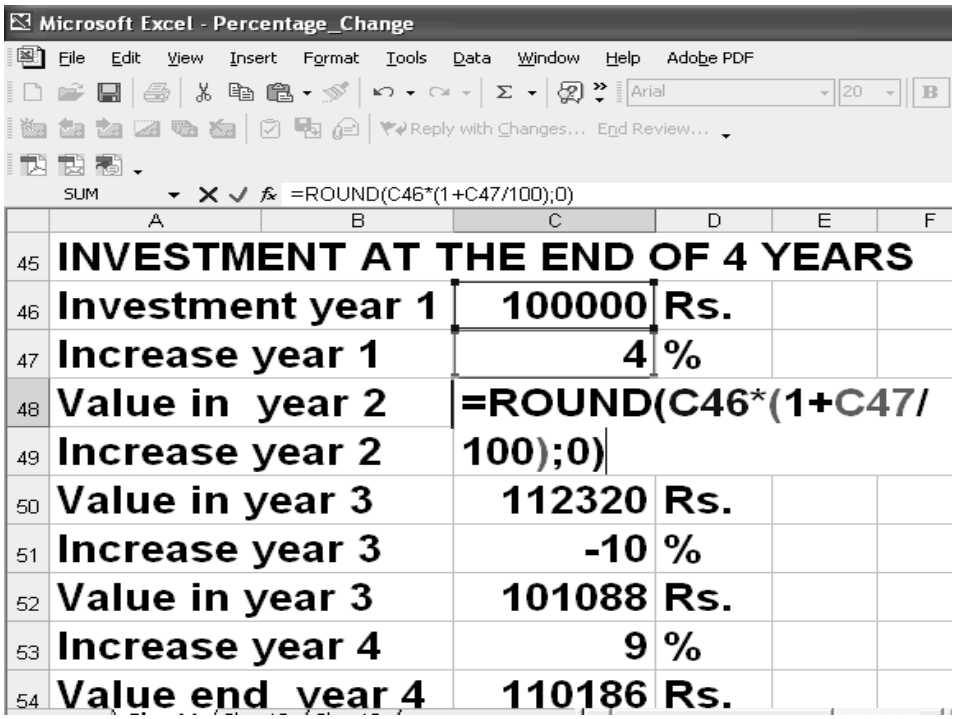
Formula for salary in year 2 in Cell C37: =ROUND(C35*(1+C36/100);0)
Formula for salary year 3 in Cell C39: =ROUND(C35*(1+C38/100);0)
Formula for salary end of year 3 in Cell C39: =ROUND(C35*(1+C39/100);0)

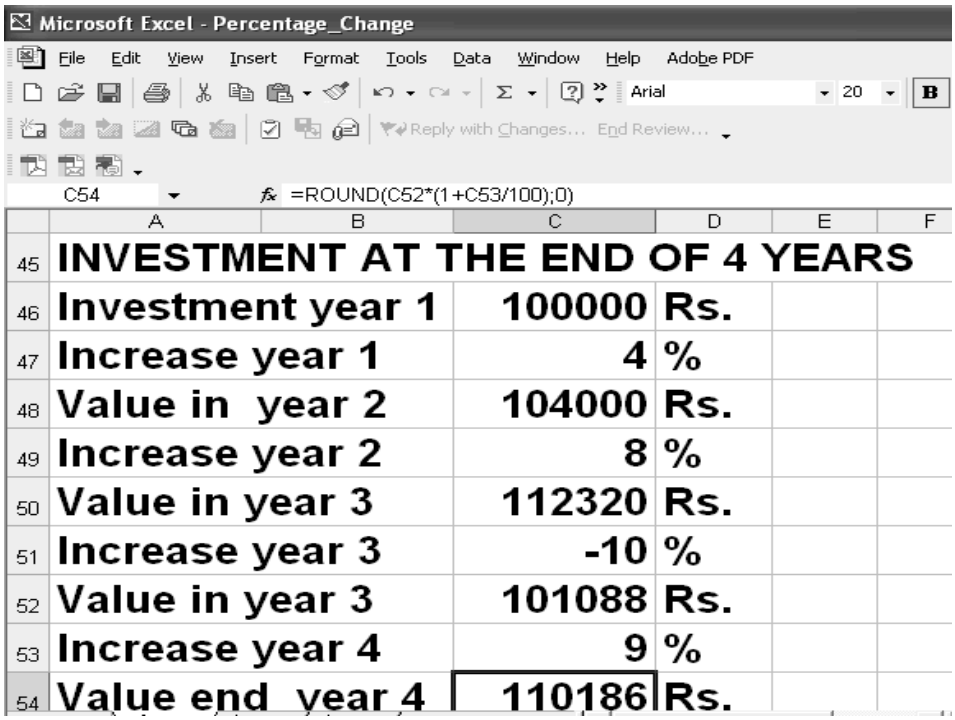
Results

Cell C37 = 5150 Rs.
Cell C39 = 5253 Rs.
Cell D22 = 5306 Rs.

EXAMPLE 5

An investment has been made for a period of 4 years.
Rates of return for each year are 4%, 8%, -10% and 9% respectively. If you invested Rs. 100,000 at the beginning of the term, how much will you have at the end of the last year?





	A	B	C	D	E	F
45	INVESTMENT AT THE END OF 4 YEARS					
46	Investment year 1		100000	Rs.		
47	Increase year 1		4	%		
48	Value in year 2		104000	Rs.		
49	Increase year 2		8	%		
50	Value in year 3		112320	Rs.		
51	Increase year 3		-10	%		
52	Value in year 3		101088	Rs.		
53	Increase year 4		9	%		
54	Value end year 4		110186	Rs.		

Calculations in Excel were done as follows:

Data entry
Cell C46: 100000
Cell C47: 4
Cell C49: 8
Cell C51: -10
Cell C53: 9

Formulas
Formula for value in year 2 in Cell C48: = ROUND(C46*(1+C47/100);0)
Formula for value in year 3 in Cell C50: = ROUND(C48*(1+C49/100);0)
Formula for value in year 4 in Cell C52: = ROUND(C50*(1+C51/100);0)
Formula for salary end of year 4 in Cell C54: = ROUND(C52*(1+C53/100);0)

Results
Cell C48 = 104000 Rs.
Cell C50 = 112320 Rs.
Cell C52 = 101088 Rs.
Cell C54 = 110186Rs.

LECTURE 6
Applications of Basic Mathematics
Part 5

OBJECTIVES

The objectives of the lecture are to learn about:

- ***Review Lecture 5***
- ***Discount***
- ***Simple and compound interest***
- ***Average due date, interest on drawings and calendar***

REVISION LECTURE 5

A chartered bank is **lowering** ***the*** **interest rate** ***on its loans*** **from** ***9%*** **to** ***7%.***

What will be the **percent decrease** ***in the interest rate on a given balance?***

A chartered bank is **increasing** ***the*** **interest rate** ***on its loans*** **from** ***7%*** **to** ***9%***

What will be the **percent increase** ***in the interest rate on a given balance?***

As we learnt in lecture 5, the calculation will be as follows:
Decrease in interest rate = 7-9 = -2 %
% decrease = -2/9 x 100 = -22.2 %
Increase in interest rate = 9-7 = 2 %
% decrease = 2/7 x 100 = 28.6 %
The calculations in Excel are shown in the following slides:

DECREASE IN RATE
Data entry
Cell F4 = 9
Cell F5 = 7

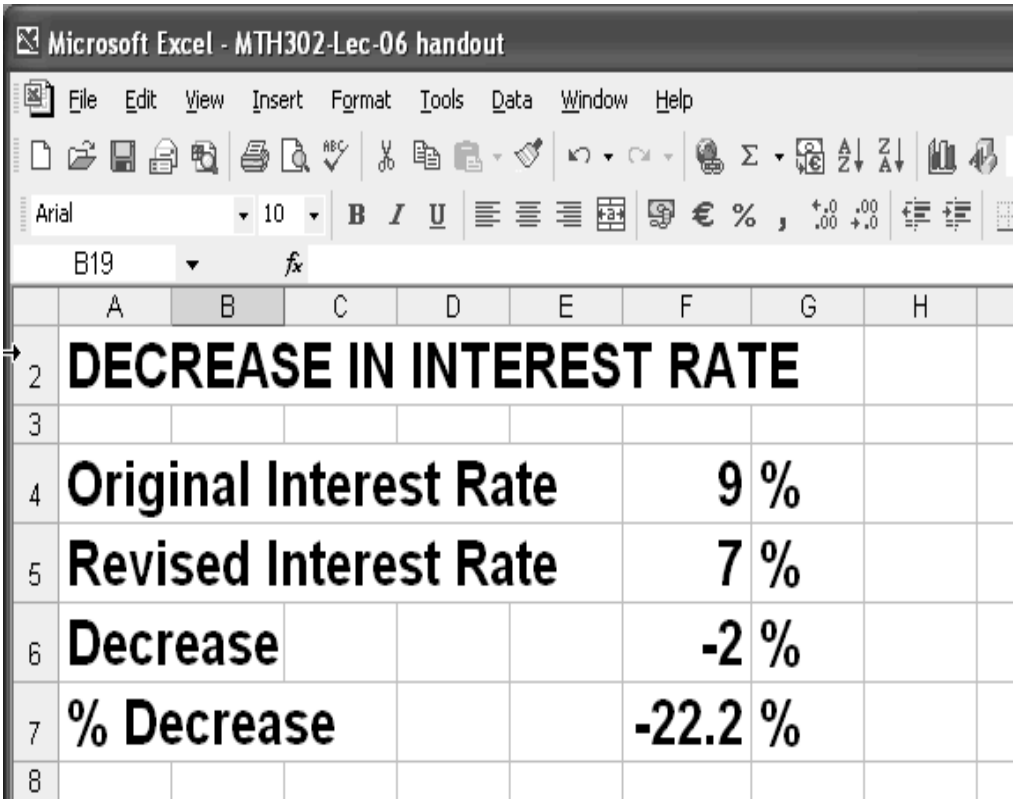
Formulas
Formula for decrease in Cell F6: = =F5-F4
Formula for % decrease in Cell F7: =F6/F4*100

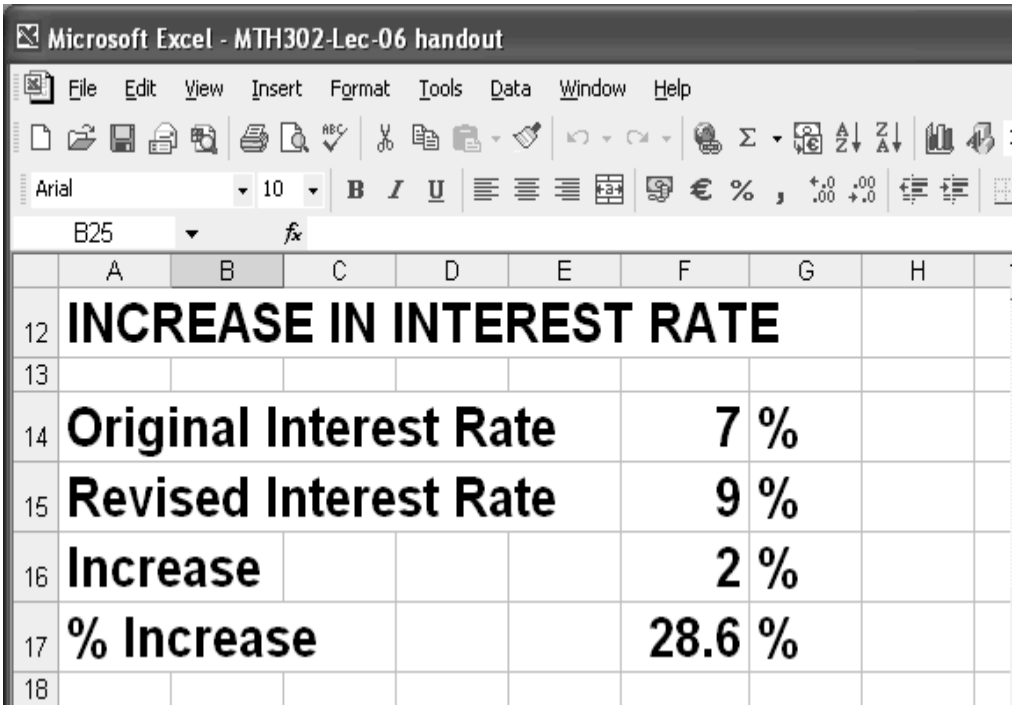
Results
Cell F6 = -2%
Cell F5 = -22.2%

INCREASE IN RATE
Data entry
Cell F14 = 7
Cell F15 = 9

Formulas
Formula for increase in Cell F16: =F15-F14
Formula for % increase in Cell F17: =F16/F14*100

Results
Cell F6 = -2%
Cell F5 = -22.2%





BUYING SHARES

If you buy 100 shares at Rs. 62.50 per share with a 2% commission, calculate your total cost.

Calculation

100 * Rs. 62.50 = Rs. 6,250
.02 * Rs. 6,250 = 125
Rs. 6,375

RETURN ON INVESTMENT

Suppose you bought 100 shares at Rs. 52.25 and sold them 1 year later at Rs. 68. With a 1% commission rate buying and selling the stock and a current Rs 10 dividend per share in effect, what was your return on investment?

Bought

100 shares at Rs. 52.25 = 5,225.00
Commission at 1% = 52.25
Total Costs = 5,277.25

Sold

100 shares at Rs. 68 = 6,800.00
Commission at 1% = - 68.00
Total Costs = 6,732.00

Gain

Net receipts = 6,732.00
Total cost = - 5,277.25
Net Gain = 1,454.75
Dividends (100*1) = 100.00
Total Gain = 1,454.75
Return on investment = 1,454.75/5277.25*100
= 27.57 %

The calculations using Excel were made as follows:

BOUGHT

Data entry

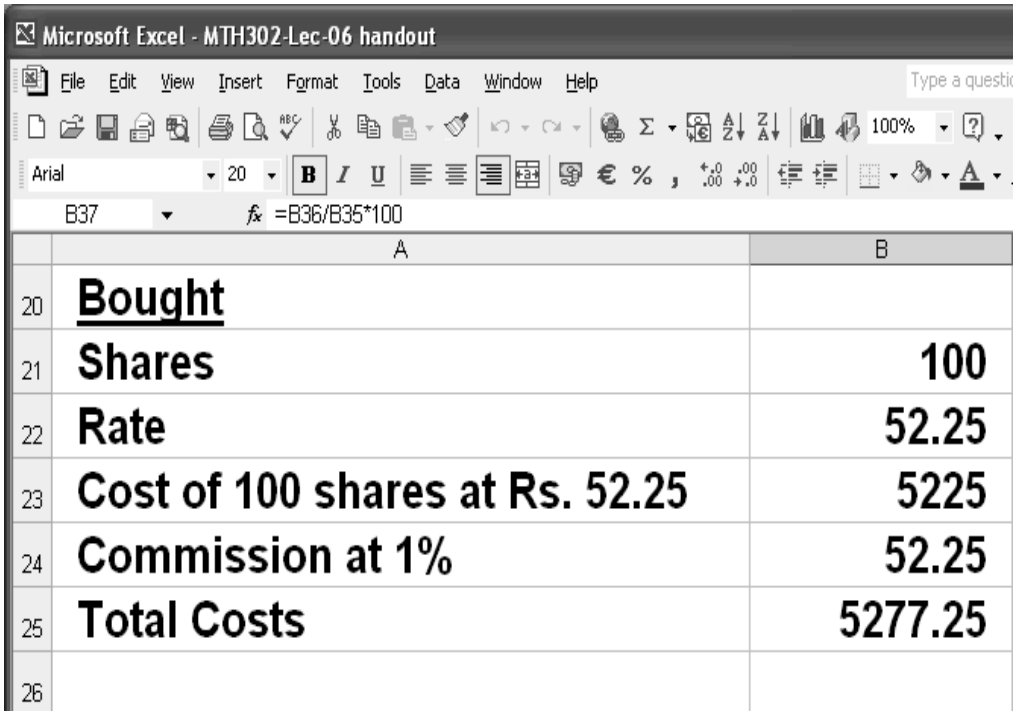
Cell B21: 100
Cell B22: 52.25

Formulas

Formula for Cost of 100 shares at Rs. 52.25 in Cell B23: =B21*B22
Formula for Commission at 1% in Cell B24: =B23*0.01
Formula for Total Costs in Cell B25: =B23+B24

Results

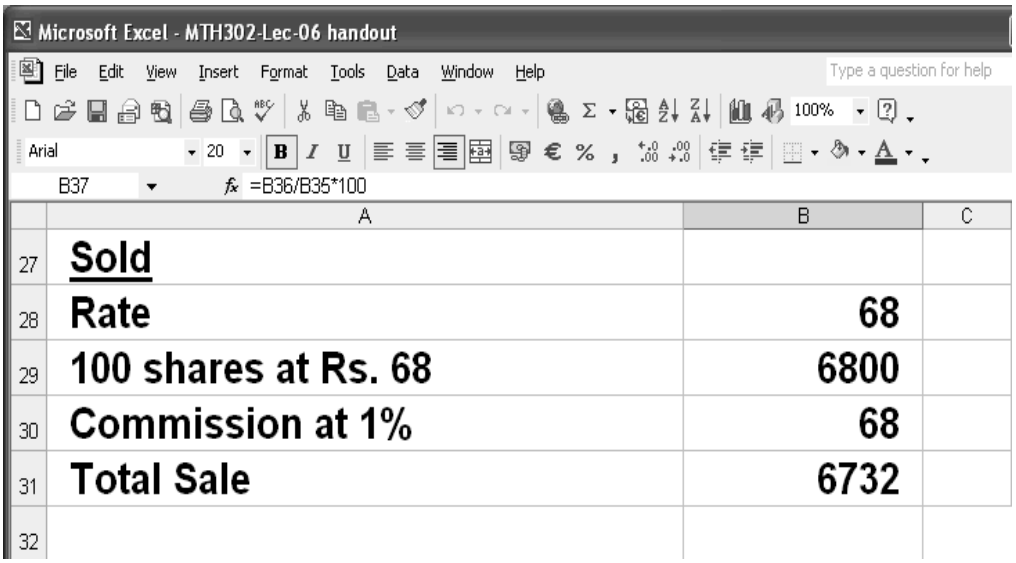
Cell B23 = 5225
Cell B24 = 52.25
Cell B25 = 5277.25



SOLD
Data entry
Cell B28: 68

Formulas
Formula for sale of 100 shares at Rs. 68 in Cell B29: =B21*B28
Formula for Commission at 1% in Cell B30: =B29*0.01
Formula for Total Sale in Cell B31: =B29-B30

Results
Cell B29 = 6800
Cell B30 = 68
Cell B31 = 6732

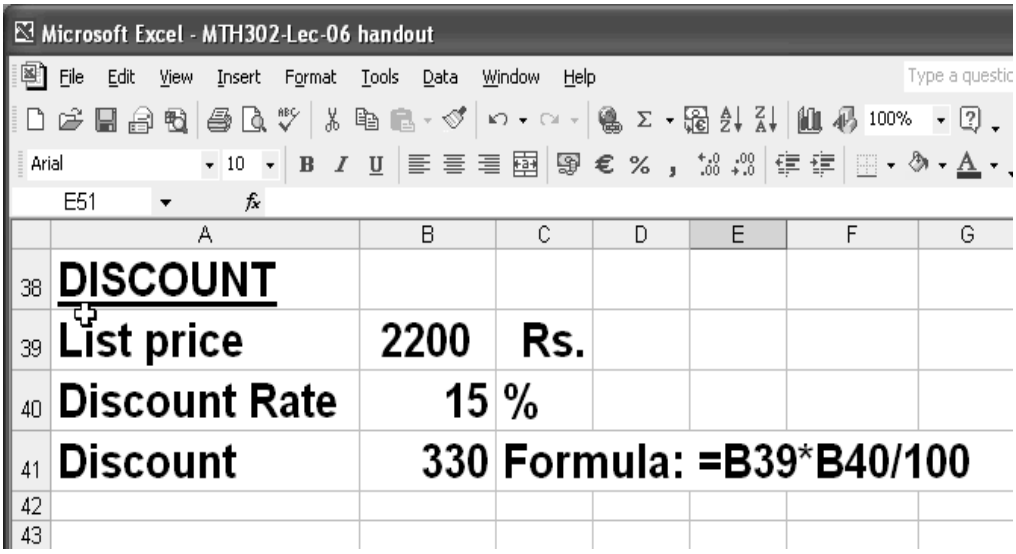


GAIN
Formulas
Formula for Net receipts in Cell B34: =B31
Formula for Total cost in Cell B35: =B25
Formula for Net Gain in Cell B36: =B31-B25
Formula for % Gain in Cell B37: =B36/B35*100

Results
Cell B34 = 6732
Cell B35 = 5277.25
Cell B36 = 1454.75
Cell B37 = 27.57

DISCOUNT
Discount is Rebate or reduction in price.
Discount is expressed as % of list price.

Example
List price = 2200
Discount Rate = 15%
Discount?
= 2200 x 0.15= 330
Calculation using Excel along with formula is given in the following slide:

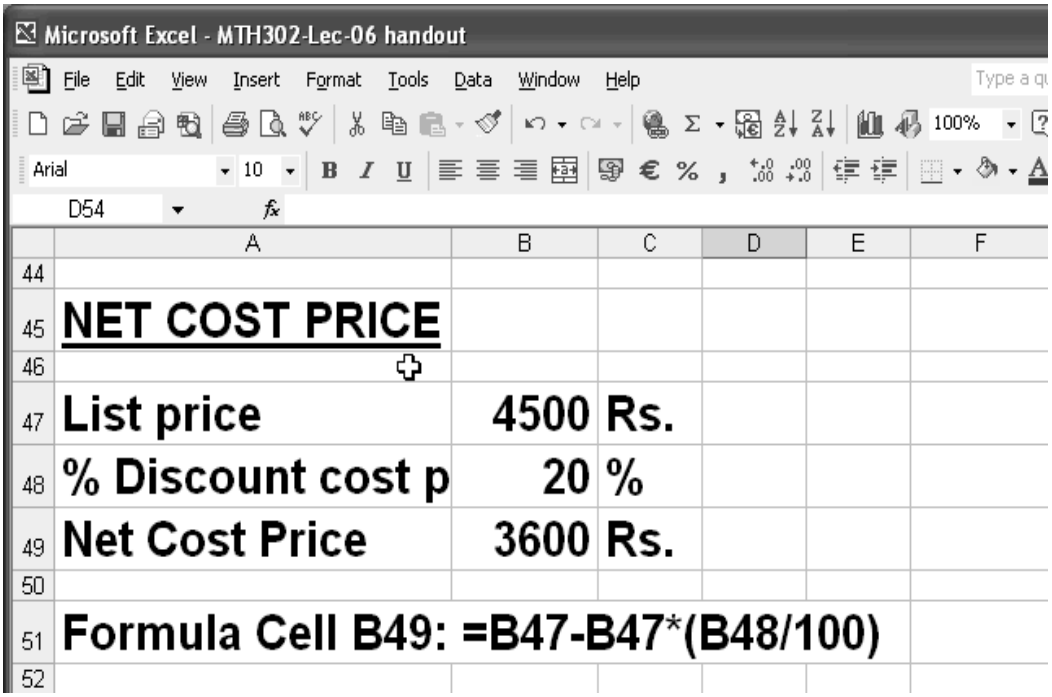


NET COST PRICE
Net Cost Price = List price - Discount

Example
List price = 4,500 Rs.
Discount = 20 %
Net cost price?

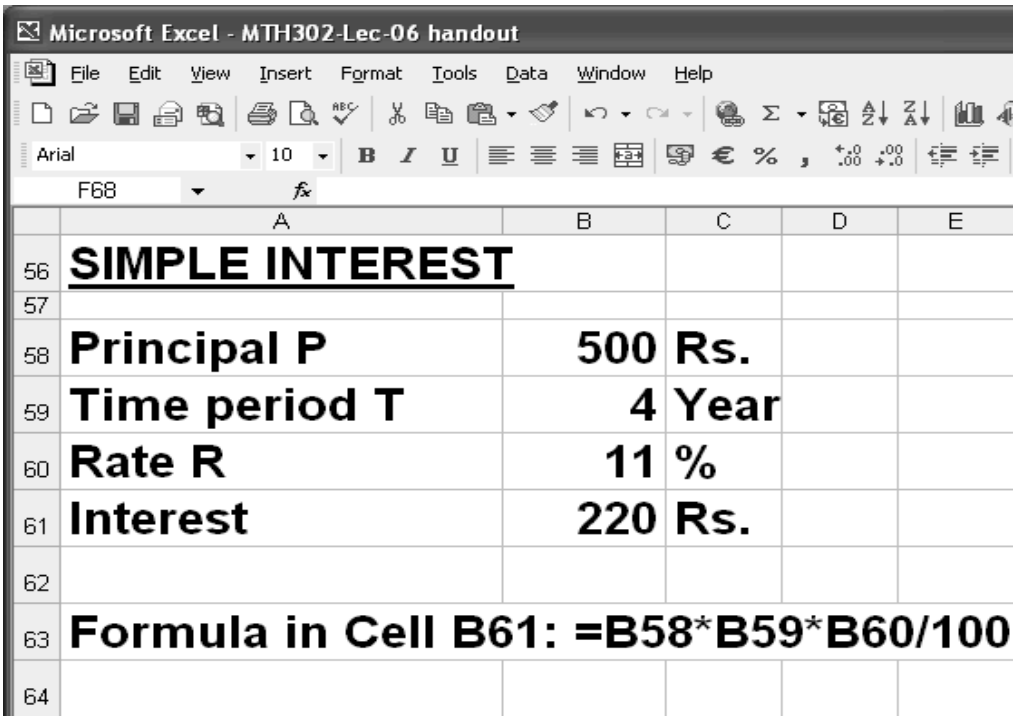
Net cost price = 4,500 – 20 % of 4,500
= 4,500 – 0.2 x4,500
=4,500 – 900
= 3,600 Rs.

Calculation using Excel along with formula is given in the following slide:



SIMPLE INTEREST
P = Principal
R = Rate percent per annum
T = Time in years
I = Simple interest
then
I = P. R. T / 100

Example
P = Rs. 500
T = 4 years
R =11%
Find interest
I = P x T x R /100
= 500 x 4 x 11/100
= Rs. 220
Calculation using Excel along with formula is given in the following slide:



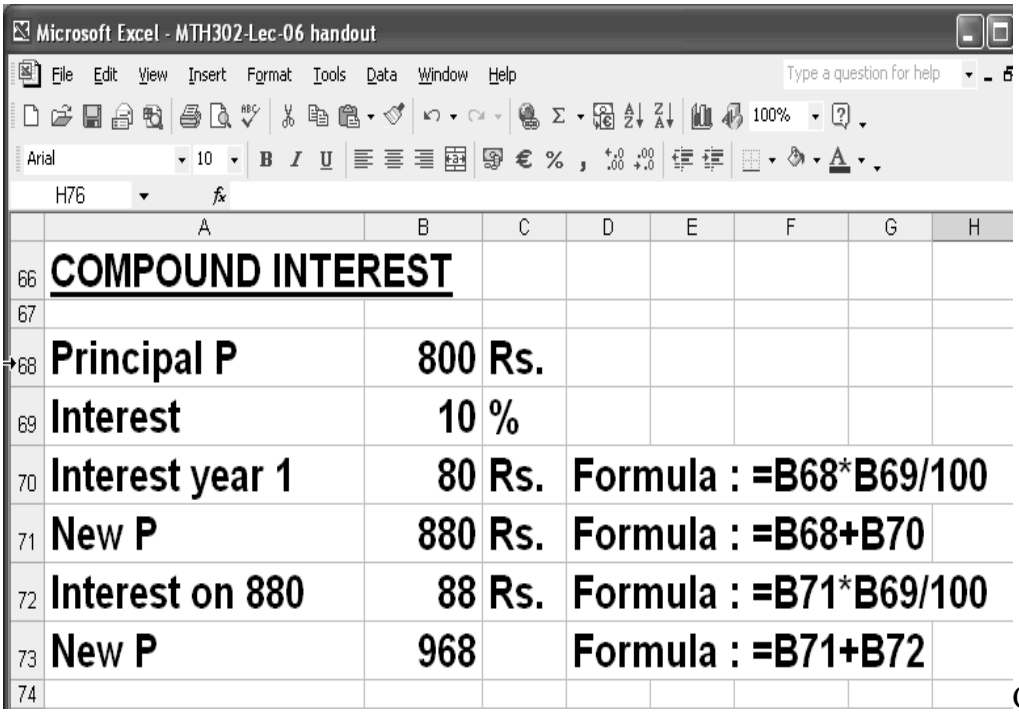
COMPOUND INTEREST

Compound Interest also attracts interest.

Example

P = 800
Interest year 1= 0.1 x 800= 80
New P = 800 + 80 = 880
Interest on 880 = 0.1 X 880 = 88
New P = 880 + 88 = 968

Calculation using Excel along with formula is given in the following slide:



Co

mpound Interest Formula

S = Money accrued after n years

P = Principal

r = Rate

n = Number of years

$S = P(1 + r/100)^{n}$

Example

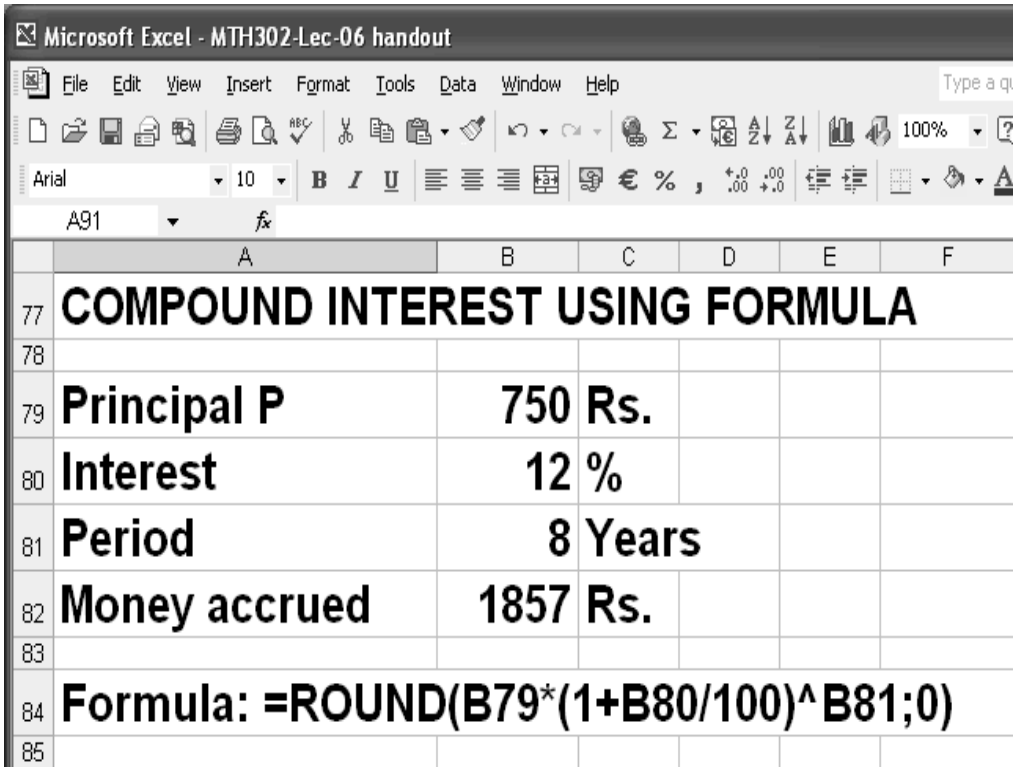
Calculate interest on Rs. 750 invested at 12% per annum for 8 years.

$S= P(1+r/100)^{8}$

$= 750(1+12/100)^{8}$

= 1957

Calculation using Excel along with formula is given in the following slide



LECTURE 7
Applications of Basic Mathematics

OBJECTIVES

The objectives of the lecture are to learn about:

- Scope of Module 2
- Review of lecture 6
- Annuity
- Accumulated value
- Accumulation Factor
- Discount Factor
- Discounted value
- Algebraic operations
- Exponents
- Solving Linear equations

Module 2

Module 2 covers the following lectures:

- Linear Equations (Lectures 7)
- Investments (Lectures 8)
- Matrices (Lecture 9)
- Ratios & Proportions and Index Numbers (Lecture 10)

Annuity

Let us look at an example to understand what is annuity. Suppose that you want to buy electric equipment on installments. The value of the equipment is Rs. 4,000. The company informs you that you must pay Rs. 1,000 at the time of purchase (down payment = 1,000). The rest of the payments are to be made in 20 installments of 200 rupees each. You are wondering about the total number and sequence of periodic payments. The sequence of payments at equal interval of time is called Annuity. The time between payments is called the Time Interval.

NOTATIONS

The following notations are used in calculations of Annuity:
R = Amount of annuity
N = Number of payments
I = Interest rater per conversion period
S = Accumulated value
A = Discounted or present worth of an annuity

ACCUMULATED VALUE

The accumulated value S of an annuity is the total payment made including the interest. The formula for Accumulated Value S is as follows:

$$S = r((1+i)^n - 1)/i$$

It may be seen that:
Accumulated value = Payment x Accumulation factor

The discounted or present worth of an annuity is the value in today's rupee value. As an example if we deposit 100 rupees and get 110 rupees (100 x 1.1) after one year, the Present Worth or 110 rupees will be 100. Here 110 will be future value of 100 at the end of year 1. The amount 110, if invested again, can be Rs. 121 after year 2. The present value of Rs. 121, at the end of year 2, will also be 100. Thus, the total present worth of payments made in year 1 and 2 (100+110 = 210) will be 200. The Future Value of this present worth is 210. (110x1.1)

DISCOUNT FACTOR AND DISCOUNTED VALUE

When future value is converted into present worth, the rate at which the

calculations are made is called Discount factor. In the previous example 10% was used to make the calculations. This rate is called Discount Rate. The present worth of future payments is called Discounted Value. The above example may be restated as follows:
The future value of Annuity in year 1 and 2 is 100 and 110 respectively. The Discount Factor is 10%. The Accumulation Factor after year 1 is 100+10/100 = 1.1. The Accumulation Factor after year 2 will be 110+11/100=1.21.
The Accumulation Factor can also be calculated by treating the value at the end of year 1 as 1 plus interest on 1. After year 1, the Accumulation Factor will be 1+0.1=1.1. Here we treated 10% of 1 as 0.1.
Obviously the Discounted Value at the beginning of year 1 can be calculated as (1+0.1)/1.1 =1. Here 1/1.1=0.9 is the Discount Factor. If you multiply the Future Value or Payment in year 2 (1.1) by the Discount Factor (0.9), you get the discounted value (1.1 x 0.9 = 1).
Thus, we can write down the formula for Discounted Value as follows:
Discounted value= Payment x Discount factor
The formula can be written as follows:
A = r ((1- 1/(1+i)^n)/i)

EXAMPLE 1. ACCUMULATION FACTOR (AF)

Calculate Accumulation Factor and Accumulated value when:
Discount rate i = 4.25 %
Number of periods n = 18
Amount of Annuity R = 10,000 Rs.
Accumulation Factor AF = ((1 + 0.0425)^18-1)= 26.24
Accumulated Value S = 10,000x 26.24 = 260,240

EXAMPLE 2. DISCOUNTED VALUE (DV)

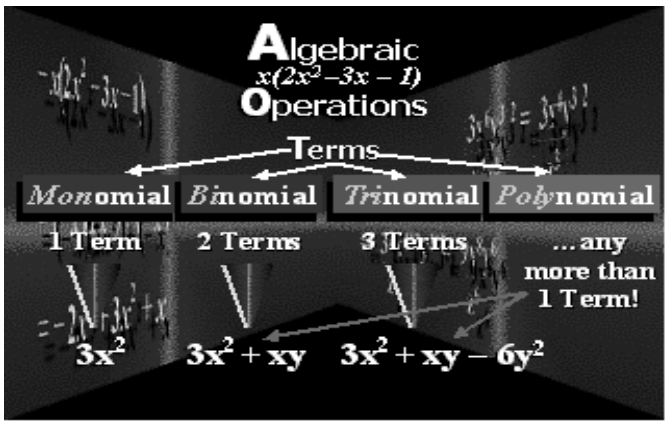
In the above example calculate the value of all payments at the beginning of term of annuity
Value of all payments at the beginning of term of Annuity
= Payment x Discount Factor (DF)
Formula for Discount Factor = ((1-1/(1+i)^n)/i)
= ((1-1/(1+0.045)^8)/0.045)
= 6.595

EXAMPLE 3. ACCUMULATED VALUE (S)

In the above example, calculate the Accumulated Value S.
ACCUMULATED VALUE
= 2,000 x ((1-1/(1+0.055)^8)/0.055)
= 2,000 x11.95
=23,900.77

ALGEBRAIC OPERATIONS

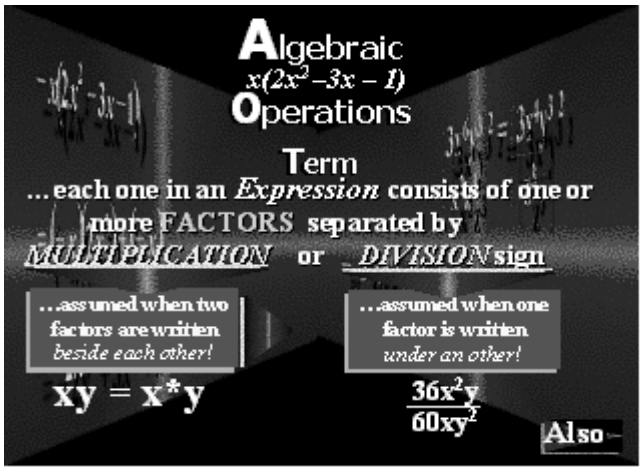
Algebraic Expression indicates the mathematical operations to be carried out on a combination of NUMBERS and VARIABLES.
The components of an algebraic expression are separated by Addition and Subtraction.
An example is the expression in the following slide. Here the components 2x^2, 3x and 1 are separated by minus "-" sign.



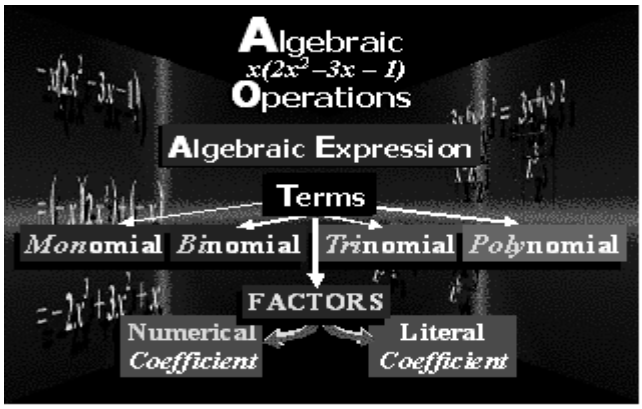
In algebraic expressions there are four types of terms:

- Monomial, i.e. 1 term (Example: 3x^2)
- Binomial, i.e. 2 terms (Example: 3x^2+xy)
- Trinomial, i.e. 3 terms (Example: 3x^2+xy-6y^2)
- Polynomial, i.e. more than 1 term (Binomial and trinomial examples are also polynomial)

Algebraic operations in an expression consist of one or more FACTORs separated by MULTIPLICATION or DIVISION sign.
Multiplication is assumed when two factors are written beside each other.
Example: xy = x*y
Division is assumed when one factor is written under an other.
Example: 36x^2y/60xy^2



Factors can be further subdivided into NUMERICAL and LITERAL coefficients.

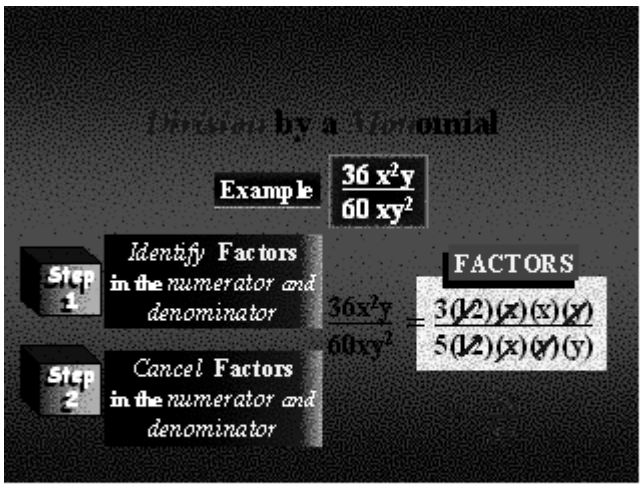


There are two steps for Division by a monomial.

1. Identify factors in the numerator and denominator
2. Cancel factors in the numerator and denominator

Example:

36x^2y/60xy^2
36 can be factored as 3 x 12.
60 can be factored as 5 x 12
x^2y can be factored as (x)(x)(y)
xy^2 can be factored as (x)(y)(y)
Thus the expression is converted to: 3 x 12(x)(x)(y)/ 5 x 12(x)(y)(y)
12x(x)(y) in both numerator and denominator cancel each other. The result is:
3(x)/5(y)

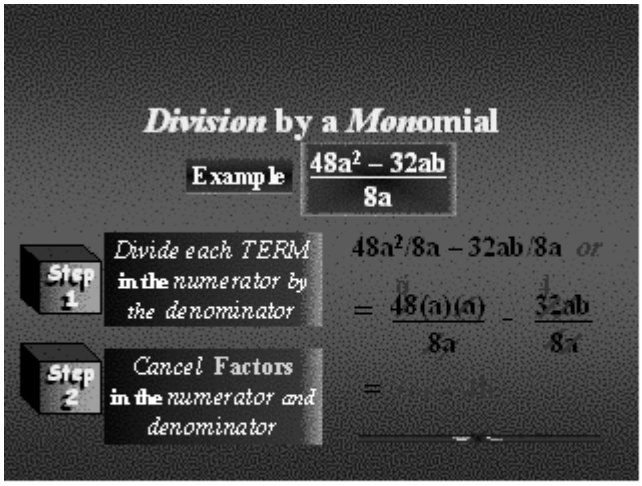


Another example of division by a monomial is (48a^2 – 32ab)/8a.

Here the steps are:

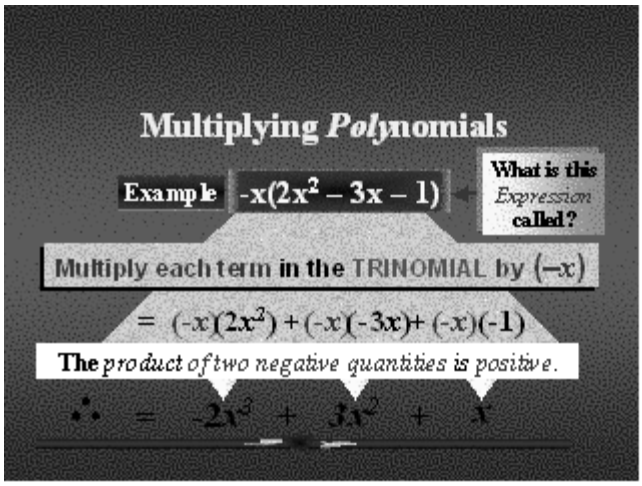
1. Divide each term in the numerator by the denominator
2. Cancel factors in the numerator and denominator

48a^2/8a = 8x6(a)(a)/8a = 6(a)
32(a)(b)/8(a) = 4x8(a)(b)/8(a) = 4(b)
The answer is 6(a) – 4(b).



How to multiply polynomials? Look at the example –x(2x^2 – 3x -1). Here each term in the trinomial 2x^2 – 3x -1 is multiplied by –x.
= (-x)(2x^2) + (-x)(-3x) + (-x)(-1)
= -2x^3 + 3x^2 + x

Please note that product of two negatives is positive.



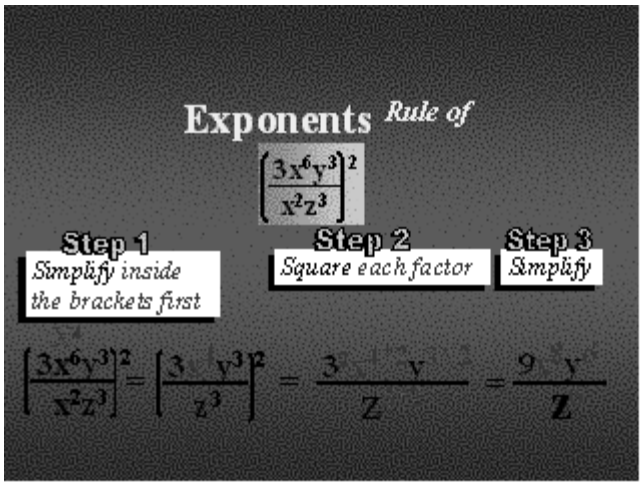
3x^6y^3/x^2z^3 Exponent of a term means calculating some power of that term. In the following example we are required to work out exponent of 3x^6y^3/x^2z^3 to the power of 2. The steps in this calculation are:

1. Simplify inside the brackets first.
2. Square each factor
3. Simplify

In the first step, the expression 3x^6y^3/x^2z^3 is first simplified to (3x^4)(y^3)/z^3.

In the next step we take squares. The resulting expression is:

(3^2)(x^4*2)(y^3*2)/z^3*2 = 9x^8y^6/z^6

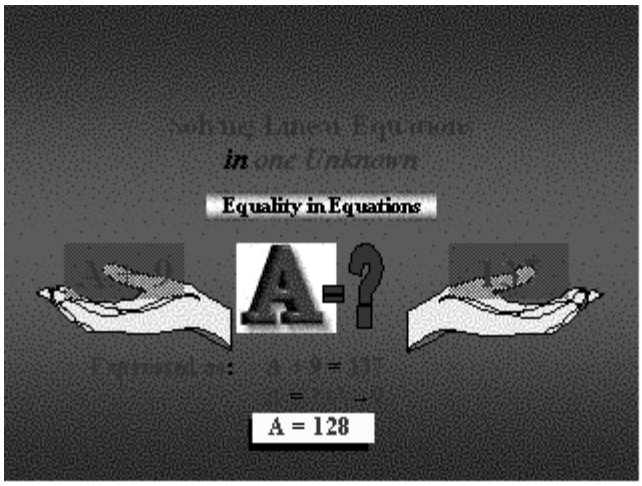


LINEAR EQUATION

If there is an expression A + 9 = 137, how do we calculate the value of A?

A = 137 – 9 = 128

As you see the term 9 was shifted to the right of the equality.



To solve linear equations:

1. Collect like terms
2. Divide both sides by numerical coefficient.

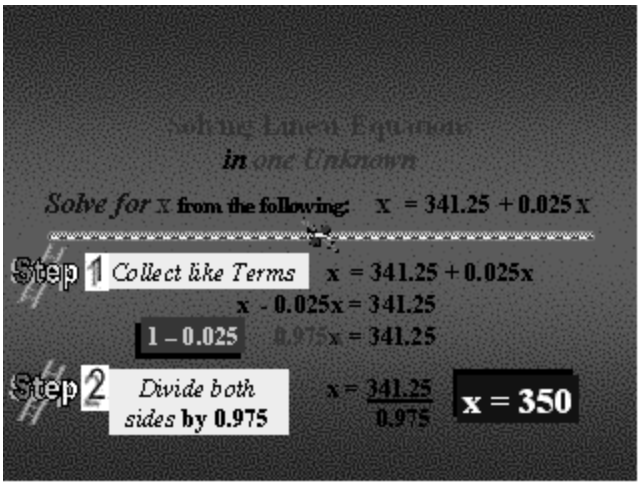
Step 1: x = 341.25 + 0.025x

x – 0.025x = 341.25

x(1-0.025) = 341.25

0.975x = 341.25

Step 2. x = 341.25/0.975 = 350



LECTURE 8

Compound Interest
Calculate returns from investments
Annuities
Excel Functions

OBJECTIVES

The objectives of the lecture are to learn about:

- Review of lecture 7
- Compound Interest
- Calculate returns from investments
- Annuities
- Excel Functions

CUMIPMT

Returns the cumulative interest paid on a loan between start_period and end_period.
If this function is not available, and returns the #NAME? error, install and load the Analysis ToolPak add-in.
The syntax is as follows:
CUMIPMT(rate,nper,pv,start_period,end_period,type)

Rate: interest rate.
Nper: total number of payment periods
Pv: present value.
Start_period: first period in the calculation
End_period: last period in the calculation
Type: timing of the payment

Type	Timing
0 (zero)	Payment at the end of the period
1	Payment at the beginning of the period

CUMIPMT-EXAMPLE

Following is an example of CUMIPMT function. In this example, in the first case the objective is to find total interest paid in the second year of payments for periods 13 to 24. Please note there are 12 periods per year. The second case is for the first payment period.

In the first formula, the Annual interest rate 9% is cell A2 (not shown here). The Years of the loan are given in cell A3. The Present value is in cell A4. For the Start period the value 13 was entered. For the End period, the value 24 has been specified. The value of Type is 0, which means that the payment will be at the end of the period. Please note that the annual interest is first divided by 12 to arrive at monthly interest. Then the Years of the loan are multiplied by 12 to get total number of months in the Term of the loan. The answer is (-11135.23).

In the second formula, which gives Interest paid in a single payment in the first month 1 was specified as the Start period. For the End period also the value 1 was entered.This is because only 1 period is under study. All other inputs were the same. The answer is (-937.50).

Data Description
9% Annual interest rate
30 Years of the loan
125,000 Present value
=**CUMIPMT**(A2/12,A3*12,A4,13,24,0)Total interest paid in the second year of payments, periods 13 through 24 (-11135.23)
=**CUMIPMT** (A2/12,A3*12,A4,1,1,0)Interest paid in a single payment in the first month (-937.50)

CUMPRINC
The CUMPRINC function returns the cumulative principal paid on a loan between two periods.
The syntax is as under:

CUMPRINC(rate,nper,pv,start_period,end_period,type)

Rate: interest rate.
Nper: total number of payment periods.
Pv: present value
Start_period: period in the calculation. Payment
End_period: last period in the calculation
Type: timing of the payment (0 or 1 as above)

CUMPRINC EXAMPLE
Following is an example of CUMPRINC function. In this example, in the first case the objective is to find the total principal paid in the second year of payments, periods 13 through 24. Please note there are 12 periods per year. The second case is for the principal paid in a single payment in the first month.
In the first formula, the Interest rate per annum 9% is in cell A2 (not shown here). The Term in years (30) is given in cell A3. The Present value is in cell A4. For the Start period the value 13 was entered. For the End period, the value 24 has been specified. The value of Type is 0, which means that the payment will be at the end of the period. Please note that the interest is first divided by 12 to arrive at monthly interest. Then the years of loan are multiplied by 12 to get total number of months in the term of the loan. The answer is (-934.1071).

In the second formula, which gives the principal paid in a single payment in the first month 1 was specified as the start period. For the end period also the value 1 was entered.This is because only 1 period is under study. All other inputs were the same. The answer is (-68.27827).

EXAMPLE
Data Description
9.00% Interest rate per annum
30 Term in years
125,000 Present value
=CUMPRINC(A2/12,A3*12,A4,13,24,0)The total principal paid in the second year of payments, periods 13 through 24 (-934.1071)
=CUMPRINC(A2/12,A3*12,A4,1,1,0)The principal paid in a single payment in the first month (-68.27827)

EFFECT
Returns the effective annual interest rate. As you see there are only two inputs, namely, the nominal interest Nominal_rate and the number of compounding periods per year Npery.

EFFECT(nominal_rate,npery)
Nominal_rate: nominal interest rate
Npery: number of compounding periods per year

EFFECT-EXAMPLE
Here Nominal_rate = 5.25% in cell A2. Npery =4 in cell A3. The answer is 0.053543 or 5.3543%. You should round off the value to 2 decimals. 5.35%.
5.25% Nominal interest rate
4 Number of compounding periods per year
=EFFECT(A2,A3)
Effective interest rate with the terms above (0.053543 or 5.3543 percent)

FV
Returns the future value of an investment. There are 5 inputs, namely, Rate the interest rate, Nper number of periods, Pmt payment per period, Pv present value and Type.
FV(rate,nper,pmt,pv,type)
Rate: interest rate per period
Nper: total number of payment periods
Pmt: payment made each period.
Pv: present value, or the lump-sum amount
Type: number 0 or 1 due

FV-EXAMPLE 1
In the formula, there are 5 inputs, namely, Rate 6% in cell A2 as the interest rate, 10 as Nper number of periods in cell A3, -200 (notice the minus sign) as Pmt payment per period in cell A4, -500 (notice the minus sign) as Pv present value in cell A4 and 1 as Type in cell A6. The answer is (2581.40).

Data	Description	EXAMPLE 1
6%	Annual interest rate	
10	Number of payments	
-200	Amount of the payment	
-500	Present value	
1	Payment is due at the beginning of the period	

=FV(A2/12, A3, A4, A5, A6)
Future value of an investment with the above terms (2581.40)
=FV(rate,nper,pmt,pv,type)

FV-EXAMPLE 2

In the formula, there are 3 inputs, namely, Rate 12% in cell A2 as the interest rate, 12 as Nper number of periods in cell A3, -1000 (notice the minus sign) as Pmt payment per period in cell A4. Pv present value and Type are not specified. Both are not required as we are calculating the Future value of the investment. The answer is (12682.50).

EXAMPLE 2

12% Annual interest rate
12 Number of payments
-1000 Amount of the payment
=FV(A2/12, A3, A4)
Future value of an investment with the above terms
(12,682.50)
FV(rate,nper,pmt,pv,type)

FV-EXAMPLE 3

In the formula, there are 4 inputs, namely, Rate 11% in cell A2 as the interest rate, 35 as Nper number of periods in cell A3, -2000 (notice the minus sign) as Pmt payment per period in cell A4, 1as Type in cell A5. The value of Pv was omitted by entering a blank for the value (note the double commas",,". The answer is (82846.25).

EXAMPLE 3

11% Annual interest rate
35 Number of payments
-2000 Amount of the payment
1 Payment is due at the beginning of the period
=FV(A2/12, A3, A4,, A5)
Future value of an investment with the above terms (82,846.25)

FV(rate,nper,pmt,pv,type)

FV SCHEDULE

Returns the future value of an initial principal after applying a series of compound interest rates.

FVSCHEDULE(principal, schedule)

Principal: present value

Schedule: an array of interest rates to apply

FV SCHEDULE-EXAMPLE

In this example, the Principal is 1. The compound rates {0.09, 0.11,0.1} are given within curly brackets. The answer is (1.33089).

FVSCHEDULE(principal,schedule)

=FVSCHEDULE(1,{0.09,0.11,0.1})

Future value of 1 with compound interest rates of 0.09,0.11,0.1 (1.33089)

IPMT

Returns the interest payment for an investment for a given period.

IPMT(rate,per,nper,pv,fv,type)

Rate: interest rate per period

Per: period to find the interest

Nper: total number of payment periods

Pv: present value, or the lump-sum amount

Fv: future value, or a cash balance

Type: number 0 or 1

ISPMT

Calculates the interest paid during a specific period of an investment

ISPMT(rate,per,nper,pv)

Rate: interest rate

Per: period

Nper: total number of payment periods

Pv: present value. For a loan, pv is the loan amount

NOMINAL

Returns the annual nominal interest rate.

NOMINAL(effect_rate,npery)

Effect_rate: effective interest rate

Npery: number of compounding periods per year

NPER

Returns the number of periods for an investment.

NPER(rate, pmt, pv, fv, type)

Rate: the interest rate per period.

Pmt: payment made each period

Pv: present value, or the lump-sum amount

Fv: future value, or a cash balance

Type: number 0 or 1 (due)

NPV

Returns the net present value of an investment based on a series of periodic cash flows and a discount rate.

PV(rate,nper,pmt,fv,type)

Rate: interest rate per period
Nper: is the total number of payment periods
Pmt: payment made each period
Fv: future value, or a cash balance Type number 0 or 1 (due)

PMT

Returns the periodic payment for an annuity.
PMT(rate,nper,pv,fv,type)
Rate: interest rate
Nper: total number of payments
Pv: present value
Fv: future value
Type: number 0 (zero) or 1

PPMT

Returns the payment on the principal for an investment for a given period.
PPMT(rate,per,nper,pv,fv,type)
Rate: interest rate per period.
Per: period and must be in the range 1 to nper
Nper: total number of payment periods
Pv: the present value
Fv: future value (0)
Type: the number 0 or 1 (due)

PV

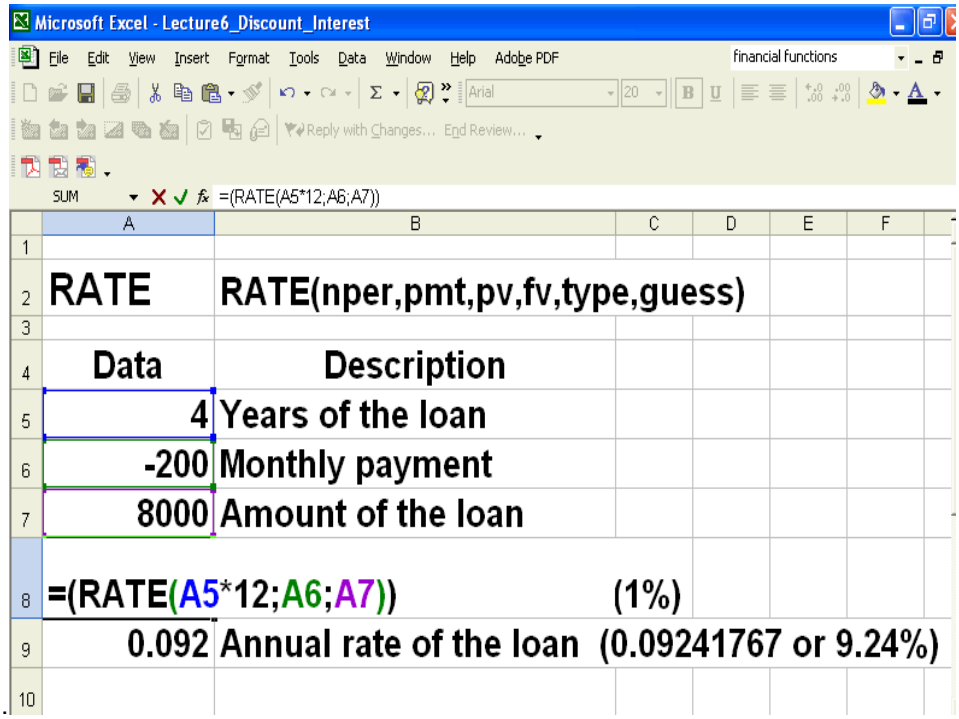
Returns the present value of an investment.
PV(rate,nper,pmt,fv,type)
Rate: interest rate per period
Nper: total number of payment periods in an annuity
Pmt: payment made each period and cannot change over the life of the annuity
Fv: future value, or a cash balance
Type: number 0 or 1 and indicates when payments are due

RATE

Returns the interest rate per period of an annuity.
RATE(nper,pmt,pv,fv,type,guess)
Nper: total number of payment periods
Pmt: payment made each period
Pv: present value
Fv: future value, or a cash balance (0)
Type: number 0 or 1 (due)
Guess: (10%)

RATE-EXAMPLE

Three inputs are specified. 4 as years of loan in cell A5, -200 as monthly payment in cell A6 and 8000 as amount of loan in cell A7. The answer is 0.09241767 or 9.24%.



LECTURE 9
Compound Interest
Calculate returns from investments
Annuities
Excel Functions

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 8
- Matrices
- Matrix Applications using Excel

QUESTIONS

Every student wonders why he or she should study matrices. Ther are mant important questions:
Where can we use Matrices?
Typical applications?
What is a Matrix?
What are Matrix operations?
Excel Matrix Functions?
There are many applications of matrices in business and industry especially where large amounts of data are processed daily.

TYPICAL APPLICATIONS

Practical questions in modern business and economic management can be answered with the help of matrix representation in:

- Econometrics
- Network Analysis
- Decision Networks
- Optimization
- Linear Programming
- Analysis of data
- Computer graphics

WHAT IS A MATRIX?

A Matrix is a rectangular array of numbers. The plural of matrix is matrices.
Matrices are usually represented with capital letters such as Matrix A, B, C.
Matrices are usually represented with capital letters.
Shown below are several matrices.

$$A = \begin{bmatrix} -1 & 9 \\ -3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 47 & 62 & 70 & 56 \\ 52 & 33 & 28 & 45 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -4 \\ 0 & 3 \\ 5 & -2 \end{bmatrix}$$

The numbers in a matrix are often arranged in a meaningful way. For example, the order for school clothing in September is illustrated in the table, as well as in the corresponding matrix.

	Size				
	Youth	S	M	L	XL
Sweat Pants	0	10	34	40	12
Sweat Shirts	18	25	29	21	7

Shorts	19	13	48	36	9
T-shirts	27	7	10	24	14

The data in the above table can be entered in the shape of a matrix as follows:

$$\begin{bmatrix} 0 & 10 & 34 & 40 & 12 \\ 18 & 25 & 29 & 21 & 7 \\ 19 & 13 & 48 & 36 & 9 \\ 27 & 7 & 10 & 24 & 14 \end{bmatrix}$$

DIMENSION

Dimension or Order of a Matrix = Number of Rows x Number of Columns

Example

Matrix T has dimensions of 2x3 or the order of matrix T is 2x3.

$$T = \begin{bmatrix} 6 & 2 & -1 \\ -4 & 0 & 7 \end{bmatrix} \begin{matrix} \leftarrow \text{row1} \\ \leftarrow \text{row2} \end{matrix}$$

$$\begin{matrix} \nearrow & \uparrow & \nwarrow \\ \text{col 1} & \text{col 2} & \text{col 3} \end{matrix}$$

ROW, COLUMN OR SQUARE MATRIX

A matrix with dimensions 1xn is referred to as a row matrix.
For example, matrix A to the right is a 1x4 row matrix.
A matrix with dimensions nx1 is referred to as a column matrix.
For example, matrix B to the right is a 2x1 column matrix.
A matrix with dimensions nxn is referred to as a square matrix.
For example, matrix C to the right is a 3x3 square matrix.

$$A = \begin{bmatrix} 12 & 17 & 10 & 9 \end{bmatrix} \quad B = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -3 & 7 \\ 6 & 8 & -1 \\ -5 & 0 & 4 \end{bmatrix}$$

ROW MATRIX

In a Row Matrix there is one row of values.
Example: In Matrix A above the dimension is 1x4.
In a Column Matrix there is one column of values.
Example: In Matrix B above the dimension is 2x1.
In a Square Matrix, there is equal number of rows and columns.
Example: In Matrix C above the dimension is 3x3.

IDENTY MATRIX

An identity matrix is a square matrix with 1's on the main diagonal from the upper left to the lower right and 0's off the main diagonal. An identity matrix is denoted as I. Some examples of identity matrices are shown below. The subscript indicates the size of the identity matrix. For example, I_n, represents an identity matrix with dimensions $n \times n$.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

MULTIPLICATIVE IDENTITY

With real numbers, the number 1 is referred to as a multiplicative identity because it has the unique property that the product a real number and 1 is that real number. In other words, 1 is called a multiplicative identity because for any real number n, $1 \bullet n = n$ and $n \bullet 1 = n$. With matrices, the identity matrix shares the same unique property as the number 1. In other words, a 2×2 identity matrix is a multiplicative inverse because for any 2×2 matrix A, $I_2 \bullet A = A$ and $A \bullet I_2 = A$

Example

Given the 2×2 matrix, $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

$$I_2 \bullet A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$$

$$A \bullet I_2 = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$$

Work

r1c1 = 1(2) + 0(-3) = 2
r2c1 = 0(2) + 1(-3) = -3
r1c1 = 2(1) + -1(0) = 2
r2c1 = -3(1) + 4(0) = -3
r1c2 = 1(-1) + 0(4) = -1
r2c2 = 0(-1) + 1(4) = 4
r1c2 = 2(0) + -1(1) = -1
r2c2 = -3(0) + 4(1) = 4

MULTIPLICATIVE INVERSES

Real Numbers

Two non-zero real numbers are multiplicative inverses of each other if their products, in both orders, is 1. Thus,

the multiplicative inverse of a real number, x is $\frac{1}{x}$ or x^{-1} since $x \bullet \frac{1}{x} = 1$ and $\frac{1}{x} \bullet x = 1$.

Example:

The multiplicative inverse of 5 is $\frac{1}{5}$ since

$5 \bullet \frac{1}{5} = 1$ and $\frac{1}{5} \bullet 5 = 1$

Matrices

Two 2×2 matrices are inverses of each other if their products, in both orders, is a 2×2 identity matrix. Thus, the multiplicative inverse of a 2×2 matrix, A is A^{-1} since $A \bullet A^{-1} = I_2$ and $A^{-1} \bullet A = I_2$

Example:

The multiplicative inverse of a matrix, $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$ is $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ since:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

LECTURE 10
MATRICES

OBJECTIVES
The objectives of the lecture are to learn about:
- **Review Lecture 9**
- **Matrices**

EXAMPLE 1
An athletic clothing company manufactures T-shirts and sweat shirts in four differents sizes, small, medium, large, and x-large. The company supplies two major universities, the U of R and the U of S. The tables below show September's clothing order for each university

University of S's September Clothing Order

	S	M	L	XL
T-shirts	100	300	500	300
sweat shirts	150	400	450	250

University of R's September Clothing Order.

	S	M	L	XL
T-shirts	60	250	400	250
sweat shirts	100	200	350	200

Matrix Representation
The above information can be given by two matrices S and R as shown below.

$$S = \begin{bmatrix} 100 & 300 & 500 & 300 \\ 150 & 400 & 450 & 250 \end{bmatrix}$$

$$R = \begin{bmatrix} 60 & 250 & 400 & 250 \\ 100 & 200 & 350 & 220 \end{bmatrix}$$

MATRIX OPERATIONS
The matrix operations can be summarized as under:

- **Organize and interpret data using matrices**
- **Use matrices in business applications**
- **Add and subtract two matrices**
- **Multiply a matrix by a scalar**

- **Multiply two matrices**
- **Interpret the meaning of the elements within a product matrix**

PRODUCTION

The clothing company production in preparation for the universities' Septmber orders is shown by the table and corresponding matrix P below.

	S	M	L	XL
T-shirts	300	700	900	500
sweat shirts	300	700	900	500

$$P = \begin{bmatrix} 300 & 700 & 900 & 500 \\ 300 & 700 & 900 & 500 \end{bmatrix}$$

ADDITION AND SUBTRACTION OF MATRICES

The sum or difference of two matrices is calculated by adding or subtracting the corresponding elements of the matrices.

To add or subtract matrices, they must have the same dimensions.

PRODUCTION REQUIREMENT

Since the U of S ordered 100 small T-shirts and the U of R ordered 60, then althogether 160 small T-shirts are required to supply both universities. Thus, to calculate the total number of T-shirts and sweat shirts required to supply both universities, add the corresponding elements of the two order matrices as shown below.

$$\begin{bmatrix} 100 & 300 & 500 & 300 \\ 150 & 400 & 450 & 250 \end{bmatrix} + \begin{bmatrix} 60 & 250 & 400 & 250 \\ 100 & 200 & 350 & 220 \end{bmatrix}$$

$$= \begin{bmatrix} 160 & 550 & 900 & 550 \\ 250 & 600 & 800 & 470 \end{bmatrix}$$

OVERPRODUCTION

Since the company produced 300 small T-shirts and the received orders for only 160 small T-shirts, then the company produced 140 small T-shirts too many. Thus, to determine the company's over-production, subtract the corresponding elements of the total order matrix from the production matrix as shown below.

$$\begin{bmatrix} 300 & 700 & 900 & 500 \\ 300 & 700 & 900 & 500 \end{bmatrix} - \begin{bmatrix} 160 & 550 & 900 & 550 \\ 250 & 600 & 800 & 470 \end{bmatrix} = \begin{bmatrix} 140 & 150 & 0 & -50 \\ 50 & 100 & 100 & 30 \end{bmatrix}$$

MULTIPLICATION OF MATRICES

To understand the reasoning behind the definition of matrix multiplication, let us consider the following example.

Competing Companies, A and B, sell juice in 591 mL, 1 L and 2 L plastic bottles at prices of Rs.1.60, Rs.2.30 and Rs.3.10, respectively. The table below summarises the sales for the two companies during the month of July.

	591mL	1L	2L
Company A	20,000	5,500	10,600
Company B	18,250	7,000	11,000

What is total revenue of Company A?
What is total revenue of Company B?
Matrices may be used to illustrate the above information.
As shown at the right, the sales can be written as a 2X3 matrix, S, the selling prices can be written as a column matrix, P, and the total revenue for each company can be expressed as a column matrix, R.

$$S = \begin{bmatrix} 20,000 & 5,500 & 10,600 \\ 18,250 & 7,000 & 11,000 \end{bmatrix} \qquad P = \begin{bmatrix} 1.60 \\ 2.30 \\ 3.10 \end{bmatrix} \qquad R = \begin{bmatrix} 77,510 \\ 79,400 \end{bmatrix}$$

Since revenue is calculated by multiplying the number of sales by the selling price, the total revenue for each company is found by taking the product of the sales matrix and the price matrix.

$$\begin{bmatrix} 20,000 & 5,500 & 10,600 \\ 18,250 & 7,000 & 11,000 \end{bmatrix} \times \begin{bmatrix} 1.60 \\ 2.30 \\ 3.10 \end{bmatrix} = \begin{bmatrix} 77,510 \\ 79,400 \end{bmatrix}$$

Consider how the first row of matrix S and the single column P lead to the first entry of R.

$$\begin{bmatrix} 20,000 & 5,500 & 10,600 \\ 18,250 & 7,000 & 11,000 \end{bmatrix} \times \begin{bmatrix} 1.60 \\ 2.30 \\ 3.10 \end{bmatrix} \Rightarrow 20,000(1.60) + 5,500(2.30) + 10,600(3.10) \Rightarrow \begin{bmatrix} 77,510 \\ 79,400 \end{bmatrix}$$

Product of First Entries; Product of Second Entries; Product of Third Entries

With the above in mind, we define the product of a row and a column to be the number obtained by multiplying corresponding entries (first by first, second by second, and so on) and adding the results.

MULTIPLICATION RULES

If matrix A is a $m \times n$ matrix and matrix B is a $n \times p$ matrix, then the product AB is the $m \times p$ matrix whose entry in the i-th row and the j-th column is the product of the i-th row of matrix A and the j-th row of matrix B.

The product of a row and a column is the number obtained by multiplying corresponding elements (first by first, second by second, and so on).

To multiply matrices, the number of columns of A must equal the number of rows of B.

MULTIPLICATION RULES

Given the matrices below, decide if the indicated product exists. And, if the product exists, determine the dimensions of the product matrix.

$$A=\begin{bmatrix}-5 & 0 & 4\\ 12 & -3 & 10\\ 7 & 15 & -1\end{bmatrix}$$

$$B=\begin{bmatrix}1 & 4\\ 9 & -1\\ -5 & 7\end{bmatrix}$$

$$C=\begin{bmatrix}4 & -3 & 1 & -7\\ 8 & -5 & -2 & 10\end{bmatrix}$$

$$D=\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}$$

$$E=\begin{bmatrix}6\\ 7\\ 8\\ 9\end{bmatrix}$$

MULTIPLICATION CHECKS

The table below gives a summary whether it is possible to multiply two matrices.
It may be noticed that the product of matrix A and matrix B is possible as the number of columns of A are equal to the number of rows of B. The product BA is not possible as the number of columns of b are not equal to rows of A.

Product	**Dimesions of the Matrices**	**Does a product exist?** (Is it possible to multiply the given matrices in this order?)	**Dimensions of Product Matrix**
AB	$A = 3\times 3 \quad B = 3\times 2$ ↑ ↑ inner dimensions	Yes, the product exists since the inner dimensions match (# of columns of *A* = # of rows of *B*).	3×2
BA	$B = 3\times 2 \quad A = 3\times 3$ ↑ ↑ inner dimensions	No, the product does not exist since the inner	n/a

		dimensions do not match (# of columns of $B \neq$ # of rows of A).	

LECTURE 11
MATRICES

OBJECTIVES
The objectives of the lecture are to learn about:
- **Review Lecture 10**
- **Matrix functions in Excel**
- **Set up and manipulate ratios.**
- **Allocate an amount on a prorata basis using proportions.**

MATRIX FUNCTIONS IN MS EXCEL

The Matrix Functions in Microsoft Excel are as follows:
MDETERM
Returns the matrix determinant of an array
MINVERSE
Returns the matrix inverse of an array
MMULT
Returns the matrix product of two arrays
MINVERSE
Returns the inverse matrix for the matrix stored in an array.
Syntax
MINVERSE(**array**)
Array is a numeric array with an equal number of rows and columns.
Remarks
- Array can be given as a cell range, such as A1:C3; as an array constant, such as {1,2,3;4,5,6;7,8,9}; or as a name for either of these.
- If any cells in array are empty or contain text, MINVERSE returns the #VALUE! error value.
- MINVERSE also returns the #VALUE! error value if array does not have an equal number of rows and columns.
- Formulas that return arrays must be entered as array formulas.
- Inverse matrices, like determinants, are generally used for solving systems of mathematical equations involving several variables. The product of a matrix and its inverse is the identity matrix — the square array in which the diagonal values equal 1, and all other values equal 0.
- As an example of how a two-row, two-column matrix is calculated, suppose that the range A1:B2 contains the letters a, b, c, and d that represent any four numbers. The following table shows the inverse of the matrix A1:B2.

	Column A	**Column B**
Row 1	d/(a*d-b*c)	b/(b*c-a*d)
Row 2	c/(b*c-a*d)	a/(a*d-b*c)

- MINVERSE is calculated with an accuracy of approximately 16 digits, which may lead to a small numeric error when the cancellation is not complete.
- Some square matrices cannot be inverted and will return the #NUM! error value with MINVERSE. The determinant for a noninvertable matrix is 0.

MINVERSE

MINVERSE(array)
Array is a numeric array with an equal number of rows and columns
Inverse of the matrix A1:B2.

	Column A	Column B
Row 1	d/(a*d-b*c)	b/(b*c-a*d)
Row2	c/(b*c-a*d)	a/(a*d-b*c)

Array formula
1. F2
2. Enter formula
3. Ctrl + Shift + Enter

MINVERSE-EXAMPLE

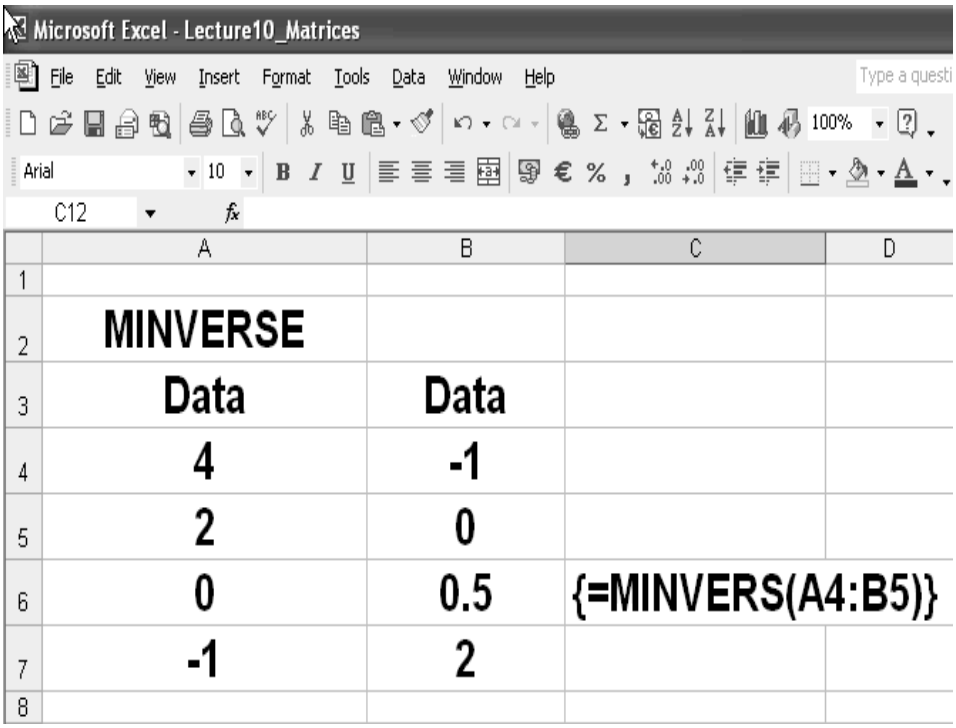
The slide below shows the inversion of matrix with row 1 [4 - 1] and row 2 [2 0].

The formula in the example must be entered as an array formula. Select the range A4:B5 starting with the formula cell. Press F2, and then press CTRL+SHIFT+ENTER. If the formula is not entered as an array formula, the single result is 0.

The process was as follows:

1. **Enter data of array to be inverted. Cells A4:B5.**
2. **Select cells A6:B7 for the formula.**
3. **Enter the formula "=MINVERSE("**
4. **Select the range A4:B5**
5. **Enter ")"**
6. **Press CTRL+SHIFT+ENTER**
7. **Press Enter**

Please note that the entry of the array Formula can be tricky. You must enter the data and formula as summarised above. If your entry is correct the curly bracket will indicate that the formula was entered as an array formula.



MDETERM

Returns the matrix determinant of an array.

Syntax

MDETERM(**array**)

Array is a numeric array with an equal number of rows and columns.

Remarks

- Array can be given as a cell range, for example, A1:C3; as an array constant, such as {1,2,3;4,5,6;7,8,9}; or as a name to either of these.
- If any cells in array are empty or contain text, MDETERM returns the #VALUE! error value.
- MDETERM also returns #VALUE! if array does not have an equal number of rows and columns.
- The matrix determinant is a number derived from the values in array. For a three-row, three-column array, A1:C3, the determinant is defined as:

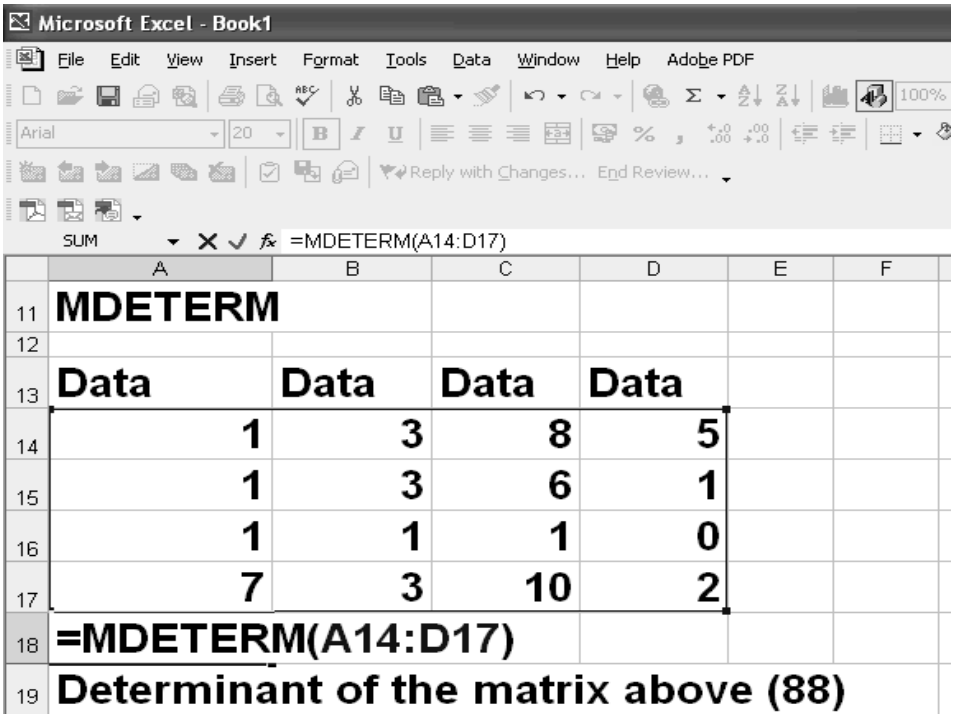
MDETERM(A1:C3) equals

A1*(B2*C3-B3*C2) + A2*(B3*C1-B1*C3) + A3*(B1*C2-B2*C1)

- Matrix determinants are generally used for solving systems of mathematical equations that involve several variables.
- MDETERM is calculated with an accuracy of approximately 16 digits, which may lead to a small numeric error when the calculation is not complete. For example, the determinant of a singular matrix may differ from zero by 1E-16.

MDETERM-EXAMPLE

The example shows an array of size 4 x 4 in cell range A14:d17. The formula was entered in cell A18. The result of this calculation is 88.



There are other ways of using this function. You can enter the matrix as an array constant.

=MDETERM({3,6,1;1,1,0;3,10,2}) Determinant of the

	matrix as an array constant (

You can calculate the determinant of the array constant.

=MDETERM({3,6;1,1})	Determinant of the matrix in the array constant (-3)

Unequal number of rows and columns results in an error.

=MDETERM({1,3,8,5;1,3,6,1})	Returns an error because the array does not have an equal number of rows and columns (#VALUE!)

MMULT

Returns the matrix product of two arrays. The result is an array with the same number of rows as array1 and the same number of columns as array2.

Syntax

MMULT(**array1**,**array2**)

Array1, array2 are the arrays you want to multiply.

Remarks

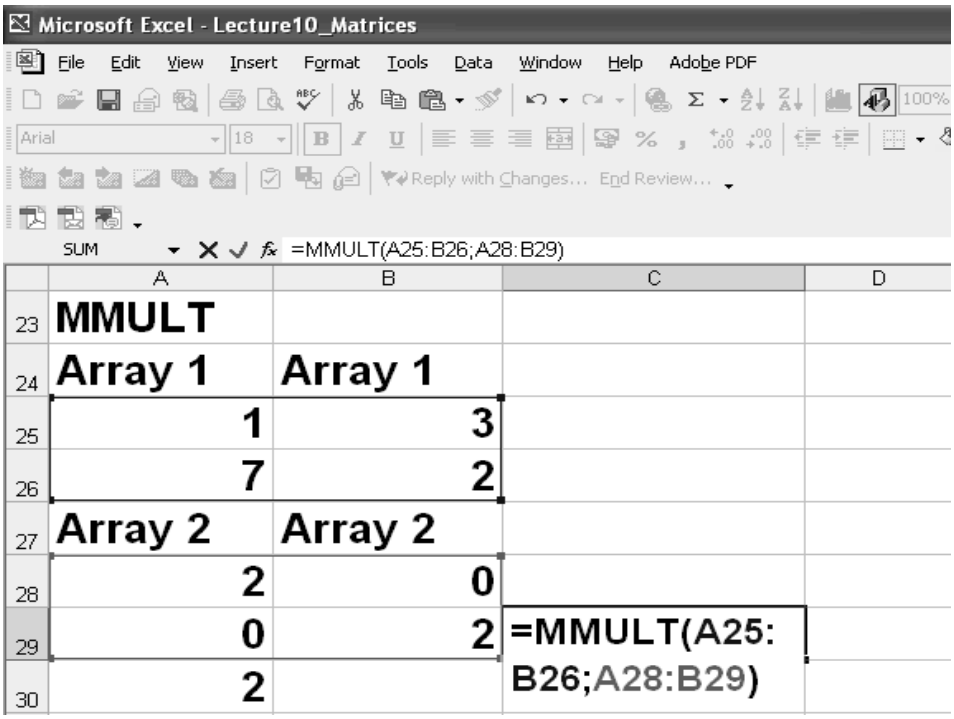
- The number of columns in array1 must be the same as the number of rows in array2, and both arrays must contain only numbers.
- Array1 and array2 can be given as cell ranges, array constants, or references.
- If any cells are empty or contain text, or if the number of columns in array1 is different from the number of rows in array2, MMULT returns the #VALUE! error value.
- The matrix product array a of two arrays b and c is:

$$a_{ij} = \sum_{k=1}^{n} b_{ik} c_{kj}$$

  where i is the row number, and j is the column number.
- Formulas that return arrays must be entered as array formulas.

MMULT-EXAMPLE

Array1 was entered in cell range A25:B26. Array2 was entered in cell range A28:B29. The



The formula was entered as an array formula. Cell A30 was selected for entry of the array Formula for MMULT. After entering "=MMULT(" the range A25:B26 was selected. Then ";" was entered. Next range A28:B29 was selected. Next ")" was entered. F2 was pressed to start the entry of array formula. Then the keys CTRL+SHIFT+ENTER were pressed simultaneously. The answer 2 was obtained in cell A30. The formula was also entered in cell C29 to show the syntax.

RATIO

A Ratio is a comparison between things. If in a room there are 30 men and 15 women then the ratio of men to women is 2 to 1. This is written as 2:1 where the ":" is the notation for a ratio.

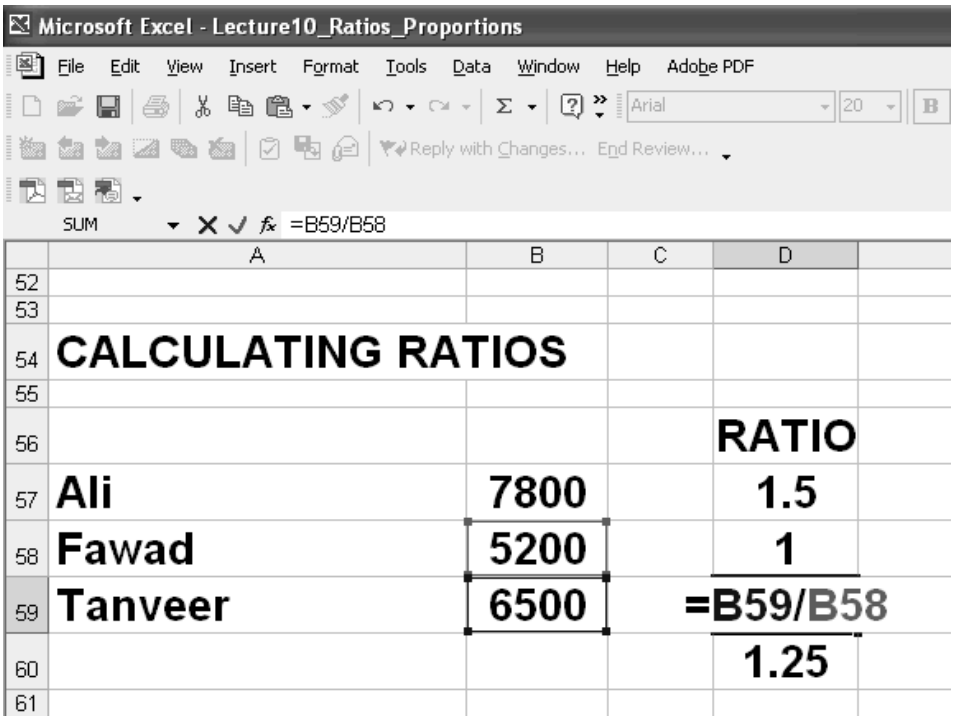
The method of calculating ratios is as under:

1. ***Find the minimum value***
2. ***Divide all the values by the smallest value.***

In the above example, the smallest value was 15. Division gives 30/15 = 2 for men and 15/15 = 1 for women. The ratio is therefore 2:1 for men and women.

RATIO-EXAMPLE

Three friends ali, Fawad and Tanveer are doing business together. To set up the business Ali invested Rs. 7800, Fawad Rs. 5,200 and Tanveer Rs. 6,500 respectively. The question is what is the ratio of their investments.
As discussed above the smallest value is 5200. All values are divided by 5200. the results are 1.5 for Ali, 1 for Fawad and 1.25 for Tanveer. The answer is:
1.5 : 1 : 1.25.



This example was solved in Excel. The formula is as under:
Cell D57: =B57/B58
Cell D58: =B58/B58
Cell D59: =B59/B58

The result for cell D59 was shown in cell D60, because the cell D59 was used to display the formula.

ESTIMATING USING RATIO

Ratio of sales of Product X to sales of Product Y is 4:3. The sales of product X is forecasted at Rs. 180,000. What should be the Sales of product Y to maintain the ratio of sales between the two products.

CALCULATION

Ratio sales X : Y = 4 : 3
Insert the value for forecasted sale for X.
180,000 : Y = 4 : 3

It can be rewritten as:

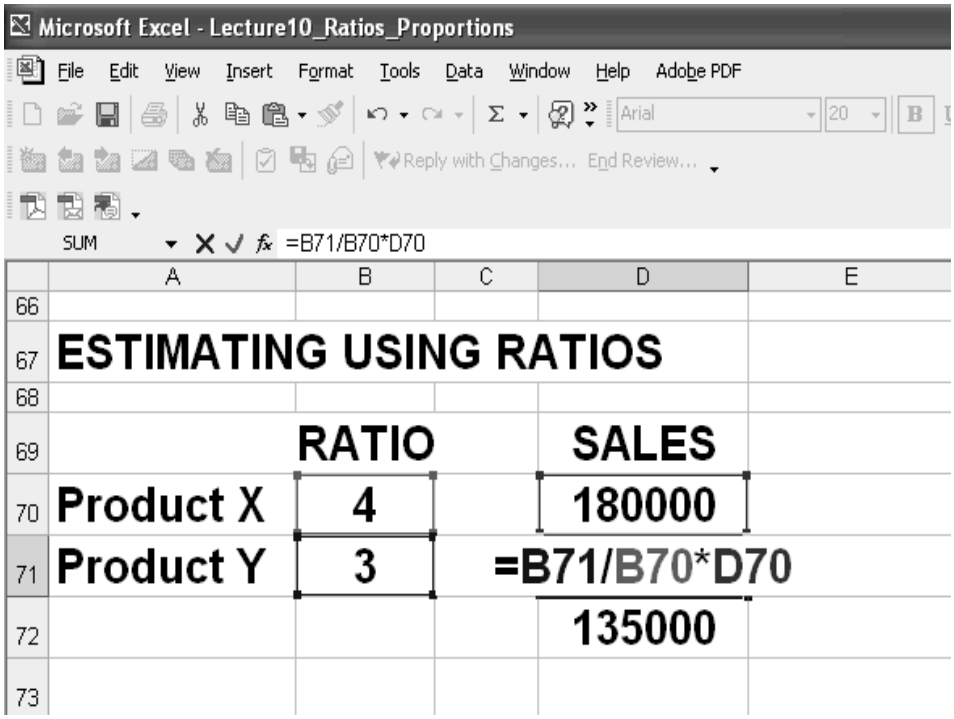
180,000/Y = 4/3

Cross – multiply
180,000 x 3 = 4 x Y
Rewrite to bring the unknown to the left of the equality
4 x Y = 180,000 x 3
Solve
Y = (180,000 x 3)/4
Y = 135,000

Calculations using EXCEL

In cells B70 and B71 the ratios of Product X and Y were entered.
The value of forecast of product X was entered in cell D70.
Before writing down the formula in excel, it was derived as follows:
1 Ratio of X = (cell B70)

2 Ratio of y = (cell B71)
3 Sale of X = (cell D70)
4 Sale of Y = (cell D71)
Now Ratio X: Y = (cell B70)/ (cell B71)
Ratio of sales = (cell D70)/ (cell D71)
Cross-multiply.
(cell B70) x (cell D71) = (cell B71) x (cell D70)
Cell D71 is unknown. Hence:
(cell D71) = (cell B71) x (cell D70)/ (cell B70)
Or
(cell D71) = (cell B71)/ (cell B70) * (cell D70)
Thus the formula was:
=B71/B70*D70
Please note that actually we are using the ratio Y to X as it is easier to think of ratio of unknown to the known.



ESTIMATING USING RATIO-EXAMPLE 2

In a 500 bed hospital there are 200 nurses and 150 other staff. If the hospital extends by a new wing for 100 beds, then what additional staff is needed?
Let us 500 beds B1 and 100 beds B2. Staff nurses N1 is 200 and other staff O1 is 150. What is the value of N2 and O2 for B2.
Obviously the ratio of beds will be used. As pointed out above, think of the ratio of unknown to known. In other words ratio B2:B1 or B2/B1. Ratio of nurses would be N2/N1. Ratio of other staff would be O2/O1.
Now :
N2/N1 = B2/B1 Or N2 = (B2/B1)*N1 or N2 = (100/500)*200 = 40 Nurses
O2/O1 = B2/B1 Or N2 = (B2/B1)*O1 or O2 = (100/500)*150 = 30 other staff.

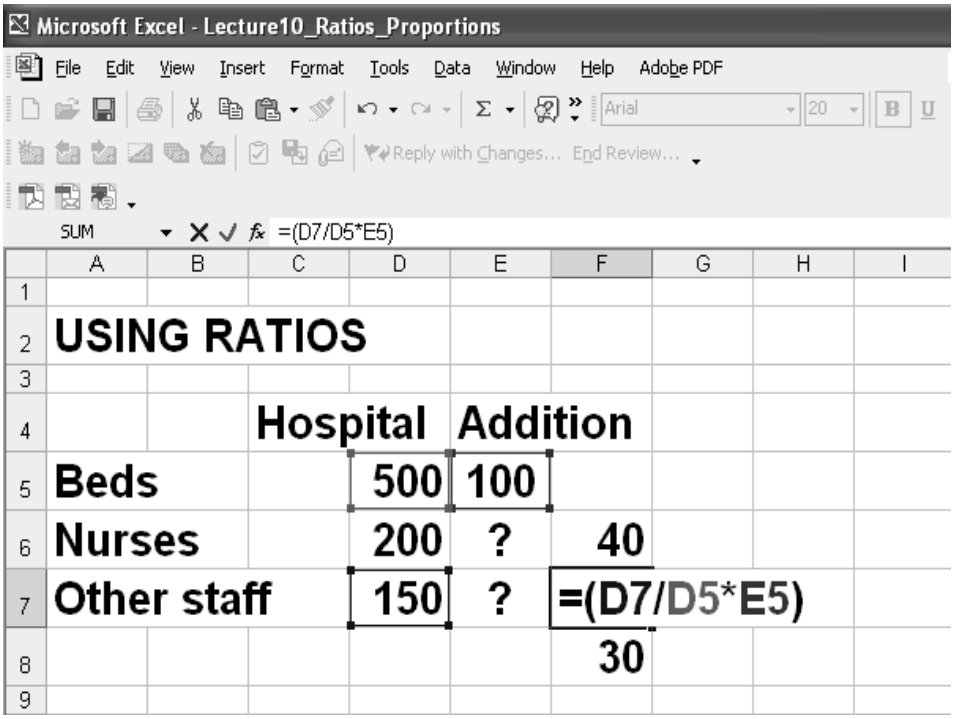
Calculation

Beds	:	Nurses	:	Other staff
500	:	200	:	150
100	:	X?	:	Y?

Nurses
500 : 200 = 100 : X
500 X = 200 x 100
X = (200 x 100)/500 = 40
Other staff
Y = (150 x 100)/500 = 30

Calculation using EXCEL

The calculation using EXCEL was done in a similar fashion as the previous example.
The calculation is self-explanatory.



ESTIMATING USING RATIO-EXAMPLE 3

A Fruit Punch recipe requires mango juice, apple juice and orange juice ratio of 3:2:1. To make 2 litres of punch calculate quantity of ingredient needed.

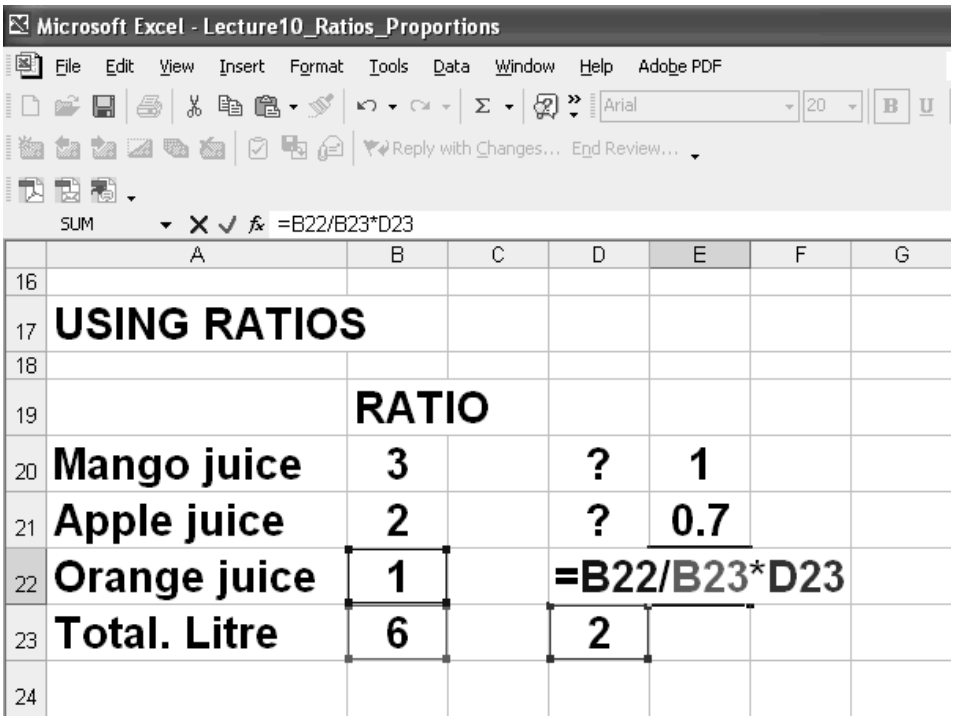
Again we shall use the raio of unknown to the unknown. The unknowns are mango and apple juice. Consider first ratio of required mango juice (3) to total quantity of punch (6). This was calculated from 3+2+1. Now the quantity of required mango for 2 litre would simply be (3/6)*2. Similarly the required quantity of apple juice is (2/6)*2.

Calculation

Mango juice : Apple juice : Orange juice
3 : 2 : 1
Total = 6
X? : Y? : Z?
Total = 2 litre
Mango juice X = (3/6)*2 = 1 litre
Apple juice Y = (2/6)*2 = 0.67 litre
Orange juice Z = (1/6)*2 = 0.33 litre

Calculation using EXCEL

Here also the similar ratios were used.
Mango = B20/B23*D23
Apple = B21/B23*D23
Orange = B22/B23*D23



LECTURE 12

RATIO AND PROPORTION
MERCHANDISING

OBJECTIVES

The objectives of the lecture are to learn about:

- Module 3
- Review Lecture 11
- Ratio and Proportions
- Merchandising
- Assignment 1A and 1B

MODULE 3

Module 3 has the following content:

- Ratio and Proportions
- Merchandising
  (Lectures 12)
- Mathematics of Merchandising
  (Lectures 13-16)

ESTIMATING USING RATIOS-EXAMPLE 1

In the previous lecture, we studied how ratios can be used to determine unknowns. Here is another example with a slightly different approach. Here, the ratios of quantities are known. Only one quantity is known. How do we estimate the total quantity that can be made? It is the quantity of orange juice that will determine the total quantity that can be made. Again the method is to use the ratio of the unknown to the known.

Punch recipe

Ratio of mango juice, apple juice and orange juice: 3:2:1. If you have 1.5 litres of orange juice, how much punch can you make?

Calculation

Mango juice : Apple juice : Orange juice

3 : 2 : 1

Total = 6

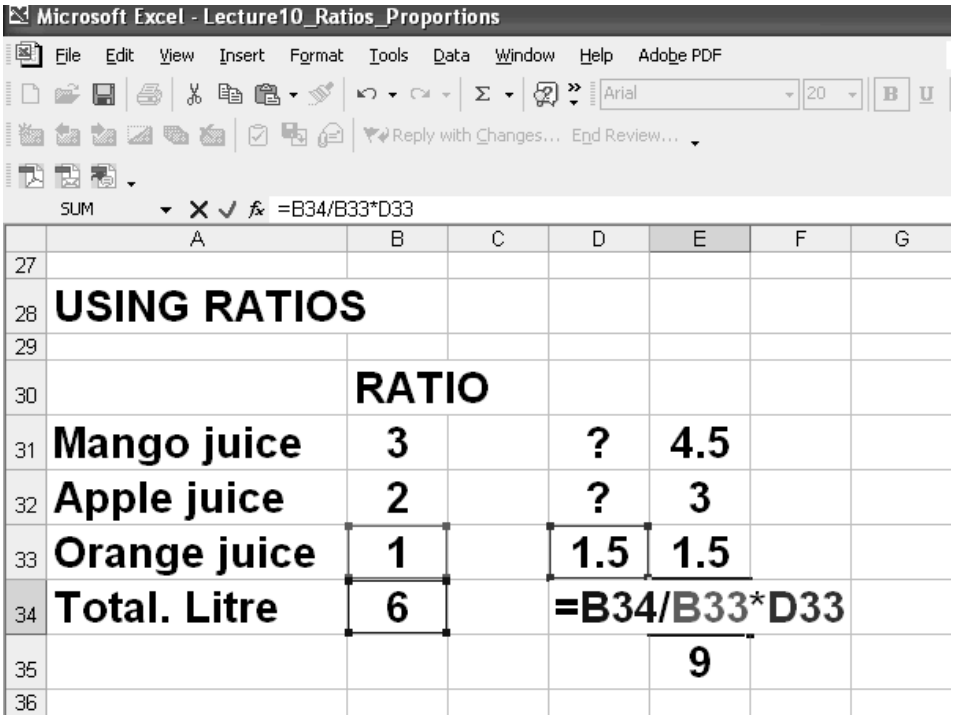
X? : Y? : Z=1.5

Total = ? litre

Mango juice X = (3/1)*1.5 = 4.5 litre
Apple juice Y = (2/1)*1.5 = 3.0 litre
Orange juice Z = 1.5 litre
Total = 4.5 + 3.0 + 1.5 = 9 litre

EXCEL Calculation

The method used is the same as used in previous examples.



ESTIMATING USING RATIOS-EXAMPLE 2

In this example, the ratios are the same. Quantity of orange juice is known. The quantity of mango and apple juice is to be calculated if the total requirement is 500 litre.

Punch recipe

The ratio of mango juice, apple juice and orange juice is 3 : 2 : 1.5 If you have 500 milliliters of orange juice, how much mango juice and apple juice is needed?

Mango juice : Apple juice : Orange juice

3 : 2 : 1.5

Total = 6.5

X? : Y? : Z = 500 litre

Total = ? litre

Mango juice X = (3/1.5)*500 = 1000 litre

Apple juice Y = (2/1.5)*500 = 667 litre

Orange juice Z = 500 litre

Total = 1000 + 667 + 500 = 2167 litre

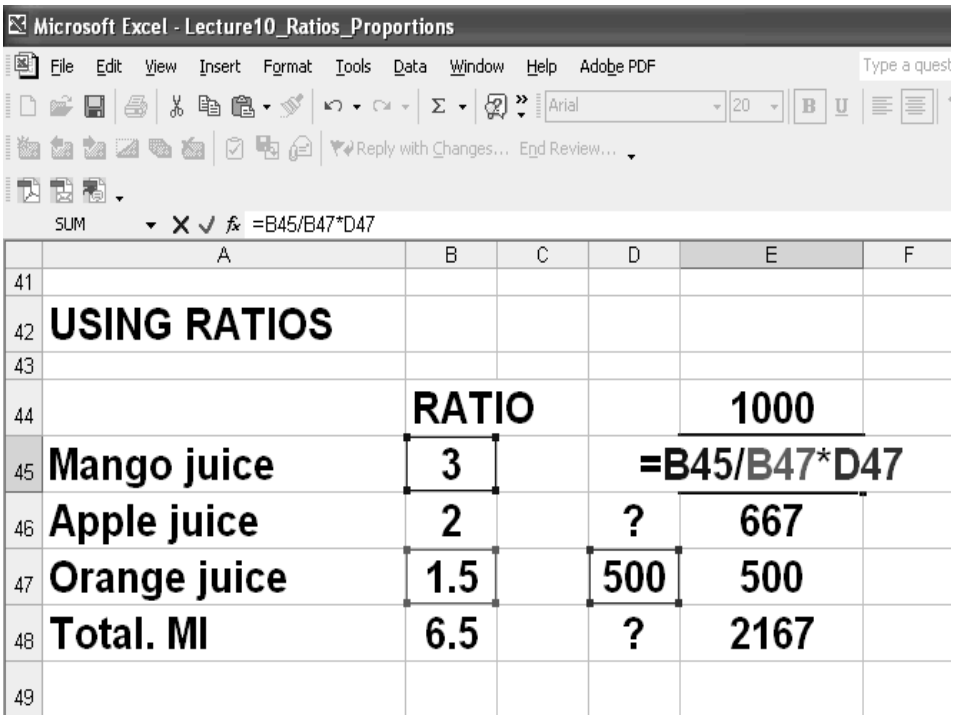
EXCEL Calculation

Here also ratios were used.

Mango = B45/B47*D47

Apple = B46/B47*D47

Orange = D47



EXERCISE

In a certain class, the ratio of passing grades to failing grades is 7 to 5. How many of the 36 students failed the course? The ratio, "7 to 5" (or 7 : 5 or 7/5), tells you that, of every 7 + 5 = 12 students, five failed. That is, 5/12 of the class flunked. Then (5/12)(36) = 15 students failed.

PROPORTION

a/b = c/d

...the values in the "b" and "c" positions are called the "means" of the proportion, while the values in the "a" and "d" positions are called the "extremes" of the proportion. A basic defining property of a proportion is that the product of the means is equal to the product of the extremes. In other words, given:

a/b = c/d

...it is a fact that ad = bc.

PROPORTION-EXAMPLES

Is 24/140 proportional to 30/176?

Check:

140×30 = 4200

24×176 = 4224

So the answer is that: They are not proportional.

PROPORTION EXAMPLE 1

Find the unknown value in the proportion: 2 : x = 3 : 9.

2 : x = 3 : 9

First, convert the colon-notation ratios to fractions:

.2/x = 3/9

Then solve:

.2/x = 3/9

18 = 3x

6 = x

PROPORTION EXAMPLE 2

Find the unknown value in the proportion: (2x + 1) : 2 = (x + 2) : 5

(2x + 1) : 2 = (x + 2) : 5
First, convert the colon-notation ratios to fractions:
(2x + 1)/2 = (x + 2)/5
Then solve:
(2x + 1)/2 = (x + 2)/5
5(2x + 1) = 2(x + 2)
10x + 5 = 2x + 4
8x = –1
x = –1/8

MERCHANDISING

What does merchandising cover?

- Understand the ordinary dating notation for the terms of payment of an invoice.
- Solve merchandise pricing problems involving mark ups and markdowns.
- Calculate the net price of an item after single or multiple trade discounts.
- Calculate a single discount rate that is equivalent to a series of multiple discounts.
- Calculate the amount of the cash discount for which a payment qualifies.

STAKEHOLDERS IN Merchandising

Who are the stakeholders in merchandising?

The main players are:

- ***Manufacturer***
- ***Middl*****emen**
- ***Receive varying levels of*** **trade discounts**
- **Retailer**
- ***Consumer***

There are discounts at all levels in the above chain.

Trade Discount

If L is the list price, then discount is calculated as % of this price. List price less discount is the net price. In mathematical terms, we can write:
Amount of discount = dL
D = Discount
L = List Price
Net Price = L(1 – d)
Net Price = List Price – Amount of Discount

LECTURE 13
MATHEMATICS OF MERCHANDISING

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 12
- Solve merchandising pricing problems involving markup and markdown

MARKUP

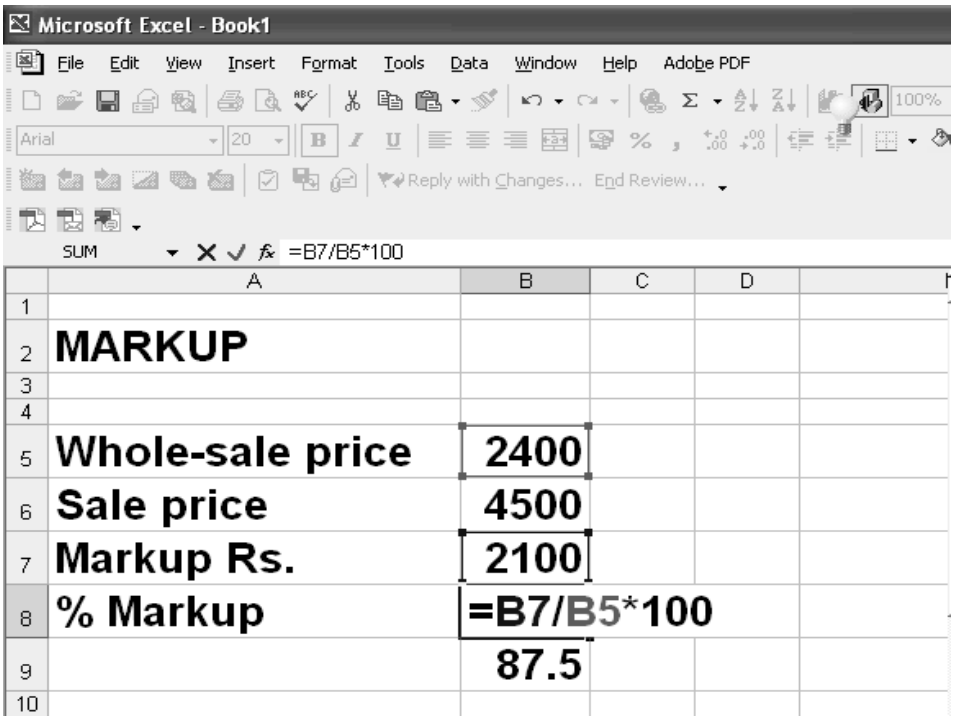
A golf shop pays its wholesaler 2 400 Rs. for a certain club, and then sells it for 4,500 Rs.
What is the markup rate?

Calculation of Markup

1. First, calculate the markup in absolute terms:
   4500 – 2400 = 2100
2. Then find the relative markup over the original price, or the markup rate:
   (2100) is (some percent) of (2400),
   or: 2100 = (x)(2400)
3. So the relative markup over the original price is:
   2100/2400 = x = 0.875
4. Since x stands for a percentage, remember to convert this decimal value to a percent.The markup rate is 87.5%.

Calculation using EXCEL

Enter whole-sale price 2400 in cell B5
Enter sale price 4500 in cell B6
Enter formula for Rs. Markup (=B6-B5) in cell B7and press enter. The answer is 2100.
Enter formula for % markup (=B7/B5*100) in cell B8 and press Enter. The answer is 87.5% shown in cell B9.

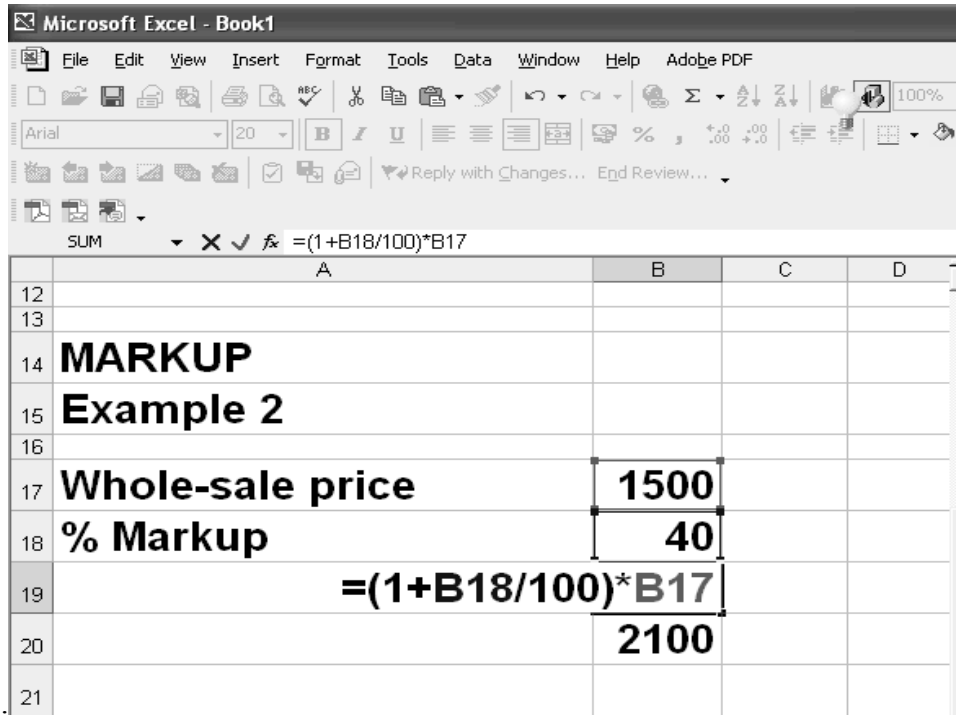


MARKUP-EXAMPLE 1
A computer software retailer used a markup rate of 40%.
Find the selling price of a computer game that cost the retailer Rs. 1,500.

Markup
The markup is 40% of the cost, so the markup is:
(0.40)(1,500) = Rs. 600

Selling Price
Then the selling price, being the cost plus markup, is:
1,500 + 600 = Rs. 2,100
The item sold for Rs. 2,100.

Calculation using EXCEL
Enter whole-sale price 1500 in cell B17.
Enter % Markup in cell B18.
Enter formula for sale price including markup (=(1+B18/100)*B17) in cell B19. Here the term 1+B18/100 is the multiplication factor. B18/100 is the markup in fraction. The result of this part of the calculation is 1.4.
The answer 2100 is shown in cell B20.
We could have calculated the multiplication factor separately. But as you see it is not necessary as the entire calculation can be done in one line



MARKDOWN
Markdown means a reduction from the original sale price. Let us look at an example to understand how markdown is calculated.

MARKDOWN-EXAMPLE 1
An item originally priced at 3,300 Rs. is marked 25% off.
What is the sale price?

Markdown

First, find the markdown.

The markdown is 25% of the original value, so:

x = (0.25)(3300) = 825

Selling Price

Then calculate the sale price, by subtracting the markdown from the original price:

3,300 – 825 = 2,475

The sale price is 2,475 Rs.

Calculation using EXCEL

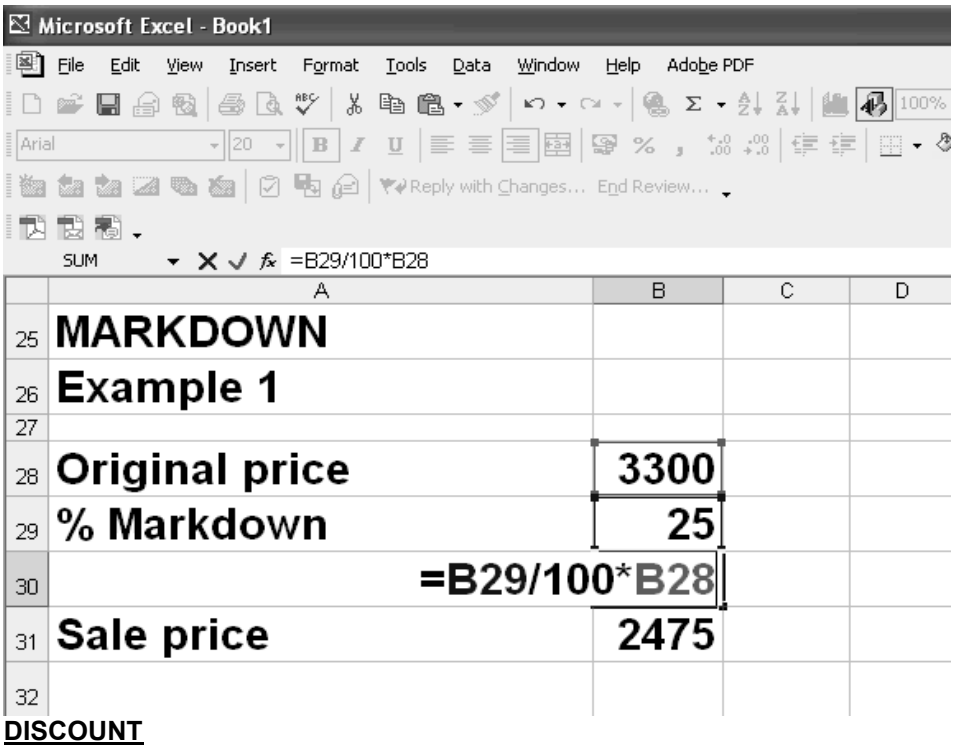
Enter original price 3300 in cell B28.

Enter % Markdown 25 in cell B29.

Enter formula for Rs. Markdown (=B29/100*B28) in cell B30. Here the term B29/100 is the markdown in fraction. The result of this part of the calculation is 825.

Enter formula for net sale price (=B28-B30) in cell B31. This formula is not shown in the slide.

We could have calculated the net sale price directly also by writing just one formula (=(1-B29/100)*B28). In other words the multiplication factor is calculated as 1-0.25 = 0.75 and multiplied with the original price 3300. The answer would be the same. By breaking the calculation in parts you can check the intermediate result and avoid errors. But if you become very conversant with formulas then you may wish to reduce the number of unnecessary steps in the calculations.



DISCOUNT

Discount is a reduction in price which the seller offers to the buyer.

DISCOUNT-EXAMPLE 1

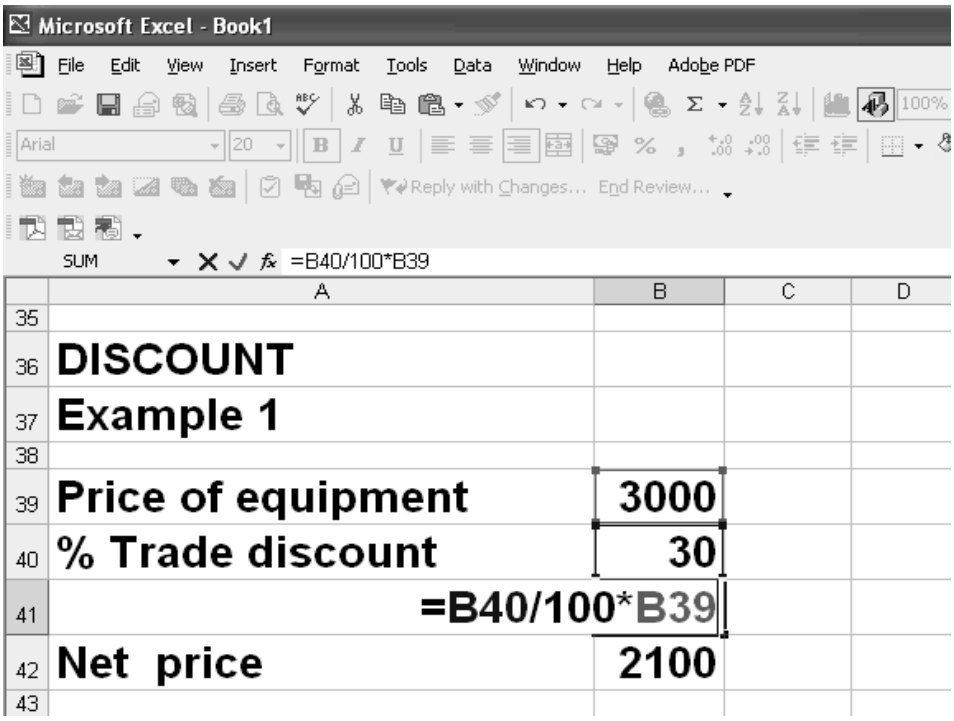
The price of office equipment is 3000. The manufacturer offers a 30% trade discount. Find the net price and the trade discount amount.

Discount

Net Price = L(1 – d)
= 3000(1 – .3)
= 3000(.7)
= 2100 Rs.
Amount of discount = dL
= .3* 3000 = 900 Rs.

Calculation using EXCEL

Enter price of equipment 3000 in cell B39.
Enter % trade Discount 30 in cell B40.
Enter formula for Rs. Discount (=B40/100*B3) in cell B41. Here the term B40/100 is the discount in fraction. The result of this part of the calculation is 900.
Enter formula for net price (=B39-B41) in cell B42. This formula is not shown in the slide. The result is 2100 as shown in cell B42.



LECTURE 14
MATHEMATICS OF MERCHANDISING
PART 2

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 13
- Financial Mathematics Part 1

SERIES DISCOUNT

This refers to the giving of further discounts as incentives for more sales

TOTAL DISCOUNT

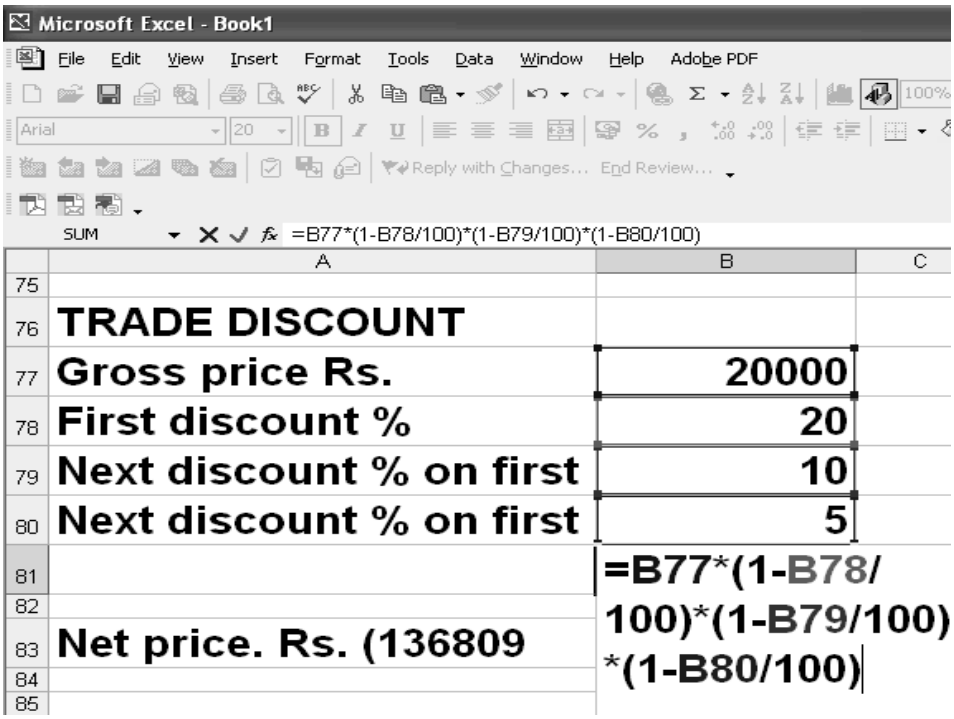
The series discount is as follows:

15% off first
Then… 10% off next
Then … 5% off next
Total discount not 30%

TRADE DISCOUNT-EXAMPLE

The price of office furniture is Rs. 20,000
The series discounts are:
20%,10%, 5%
What is the net price?
Trade Discount
$N = L(1 – d)$
$N = (1-d1)\ (1-d2)\ (1-d2)$
$= 20,000(1-.2)(1-.10)(1-.05)$
$= 20,000(.8)(.9)(.95)$
$= 20,000(.6840)$
= 13,680 Rs.

The calculation = 20,000(1-.2)(1-.10)(1-.05) was used to write the formula for the discount. Then the discount was subtracted from the original price.



LIST PRICE

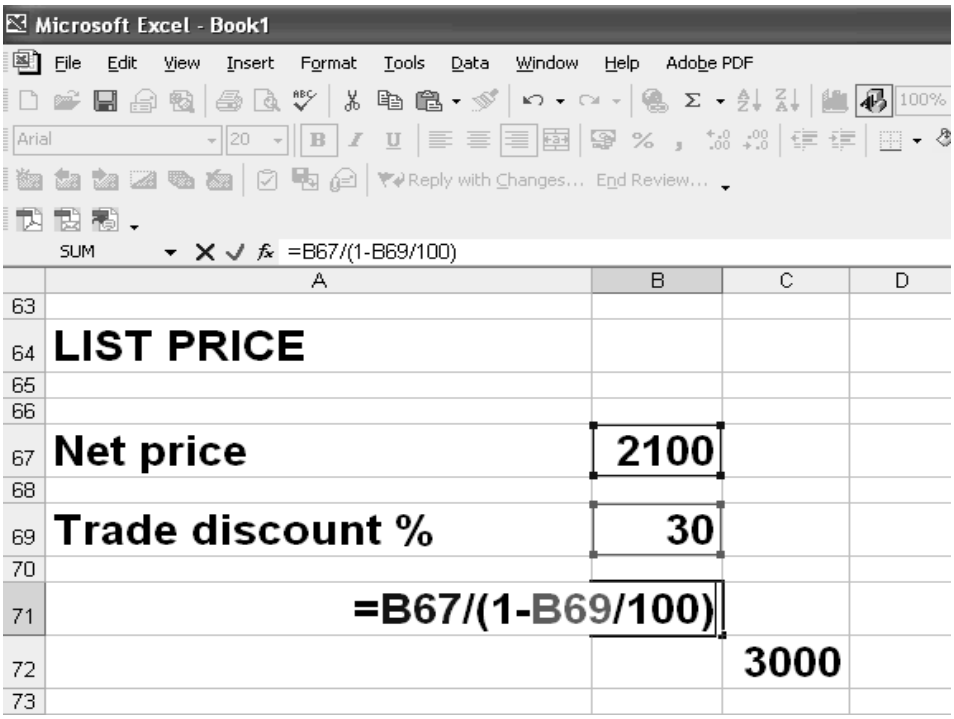
An order for power tools has a Rs. 2100 net price after a 30% trade discount. What is the list price?

Net Price

Net Price = L(1 – d)
L = N/(1 – d)
= 2100/(1 – .3)
= 2100 /(.7)
= 3000 Rs.

EXCEL Calculation

EXCEL formula for original price was based on the calculation = 2100/(1 – .3).
The net price was entered in cell B67.
% Trade discount was entered in B69.
The formula for Original price was entered in cell B71 as =B67/(1-B69/100).
The answer is shown in cell C72 as 3000.



TRADE DISCOUNT-EXAMPLE 2

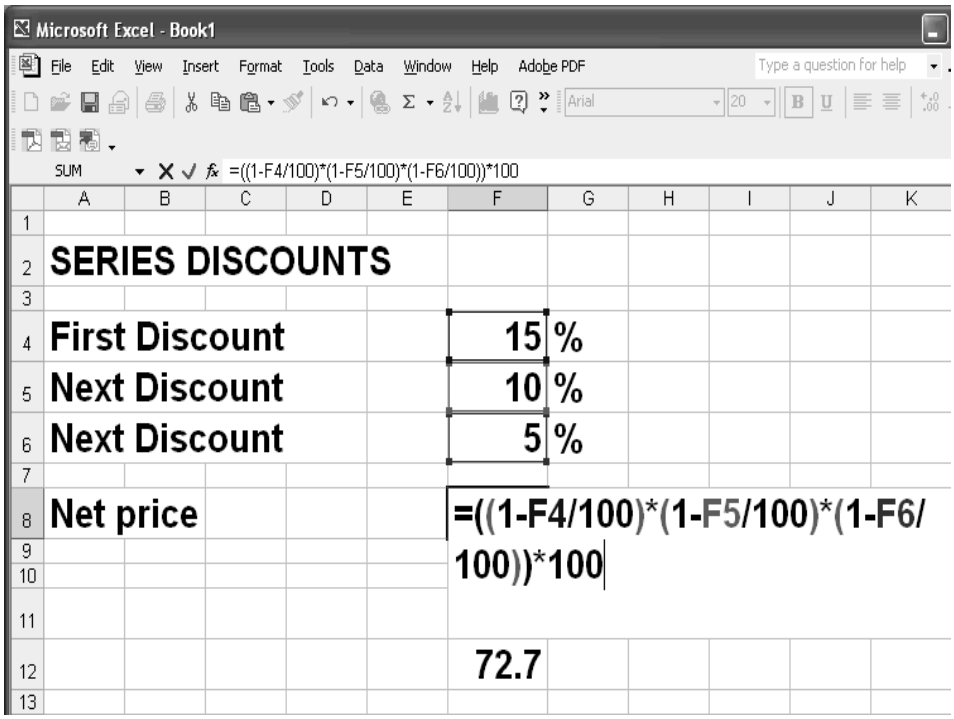
Find the single discount rate that is equivalent to the series
15%, 10% and 5%.
Net Price N = L(1 – d)

Trade Discount

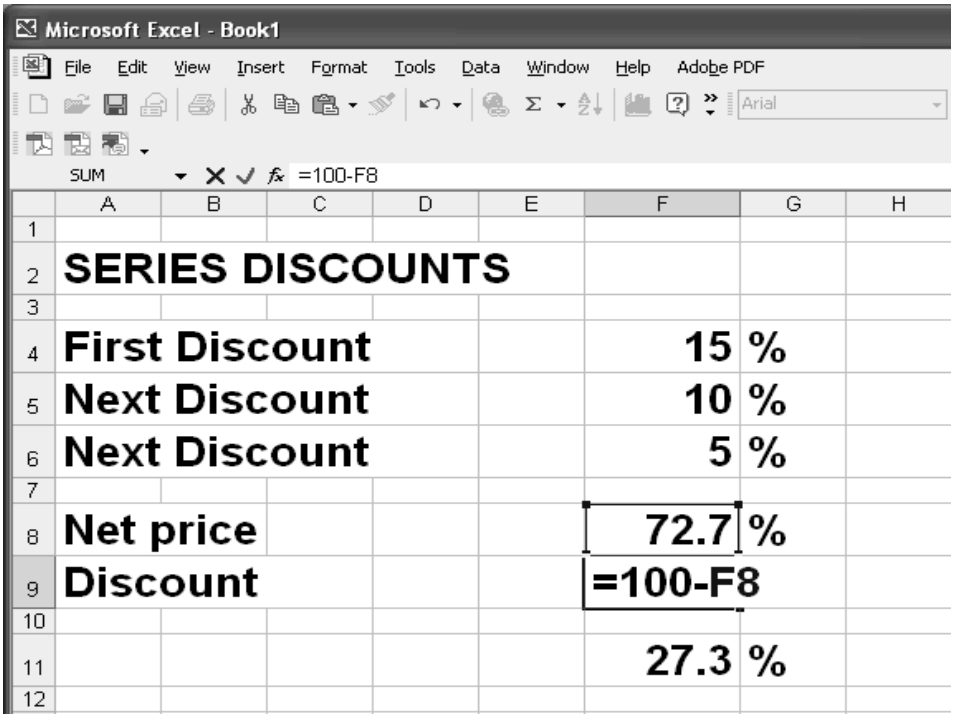
Apply the multiple discounts to a list price of Rs. 100.
(1-d1)(1-d2)(1-d3)
15%, 10%, 5%
=100(0.85)(0.9)(0.95)
= 100(0.7268)
= 72.68%
% Discount = 100 - 72.68%
= 27.62%

EXCEL Calculation

EXCEL formula for original price was based on the calculation =100(0.85)(0.9)(0.95).
The formula for net price was entered in cell F8.
The formula is shown in cell F8.
The answer is shown in cell F12.



In the following slide, the net price was calaculated in cell F8. Then, the discount was calculated assuming the original price was 100. This is a common method to assume 100 as the price when no price is given but you are required to calculate the net discount.



TRADE DISCOUNT-EXAMPLE 3

The price of car parts is Rs. 20,000.
The series discounts are 20%, 8%, 2%.
What is the single equivalent discount rate?

Trade Discount

$=100(0.8)(0.92)(0.98)$
= 100(0.7213)
= 72.13%
% Discount = 100 - 72.13%
= 27.87%
Rs. Discount = (0.2787)(20000)
= 5,574 Rs.

EXCEL Calculation

EXCEL formula for net price was based on the calculation =100(0.8)(0.92)(0.98).
The formula for net price was entered in cell F21.
The formula is not shown.
Price of car parts was entered in cell F23.
Formula for discount was based on (0.2787)(20000) and is shown in cell F24.
The answer is shown in cell F26 as 5574.

LECTURE 15
MATHEMATICS OF MERCHANDISING
PART 3

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 14
- Financial Mathematics Part 2

PARTIAL PAYMENTS

When you buy on credit and have cash discount terms, part of the invoice may be paid within the specified time. These part payments are called Partial Payments.
Let us look at an example:
You owe Rs. 40,000.
Your terms were 3/10 (3% discount by $10^{th\ day}$).
Within 10 days you sent in a payment of Rs. 10,000.
Rs. 10,000 was a part payment.
How much is your new balance?

MARKETING TERMS

There are a number of marketing terms.
First of these is the Manufacturer Cost. This is the cost of manufacturing.
Next is the price charged to middlemen in 'The Distribution Chain'.
The Distributor>Wholesaler>Retailer is a chain.
The next term is the Selling Price. This is the price charged to Consumers by Retailers.

MARKETING, OPERATING EXPENSES AND SELLING PRICE

Gross Sales less Cost of Goods sold gives the Gross Profit. The gross Profit less the Operating Expenses gives the Net Profit.

Marketing

Gross Sales	Rs	X	
Less: Cost of Good Sold		X	
Gross Profit		X	(Margin/Markup)
Less: Operating Expenses		X	
Net Profit (Income)	Rs.	X	

Operating Expenses

Expenses the company incurs in operating the business, e.g. rent, wages and utilities is called operating Expenses.

Selling Price

Selling Price is composed of Cost and Markup.
Selling Price (S) = Cost (C) + Markup (M)

MARGIN

While determining Sale Price, a company includes the operating expenses and profit to their own cost. This amount is called the margin of the company.

Example

A computer's cost is 9000. An amount of Rs. 3,000 was added to this cost by the retailer to determine the sale price for the consumer.
Thus, the selling price

= 9,000 + 3,000 Rs.
= 12,000 Rs.
Rs. 3,000 is Margin available to meet Expenses and make a Profit.

MARKUP

If the Markup is to be 33% on Cost then…
Selling Price (S) = Cost (C) + Markup (M)
133% = 100% + 33%
Cost is 100% the Base.
% Markup is the Rate.
Rs. Markup is the Portion.

MARKUP-EXAMPLE

You buy candles for Rs. 10.
You plan to sell them for Rs.15.
What is your Rs. Markup?
What is your percent Markup on cost?
Selling price – Cost = 15 – 10
= Markup = Rs. 5
% Markup = 5/10*100 = 50%

SELLING PRICE

Fawad's Appliances bought a sewing machine for Rs. 1,500.
To make the desired profit, he needs a 60% Markup on Cost.
What is Fawad's Rs. Markup?
What is his Selling price?

Selling Price

Rs. Markup = 1,500 x 0.6 = 900 Rs.
Selling Price = 1,500 + 900 = 2,400 Rs.
Or
= 1,500 x (1+0.6)
= 1,500 x 1.6
= 2,400 Rs.

EXCEL Calculation

Here 1,500 is the Sewing machine cost in cell F4 and 0.6 is the Percent Rs. Markup on cost in cell F5.
EXCEL formula in cell F6 for Rs. Markup on Cost was based on the calculation = 1,500 x 0.6.
The Selling price was calculated in cell F7 by using the formula =F4+F6.
The answer as shown in cell F7was 2400.

RS. MARKUP AND PERCENT ON COST

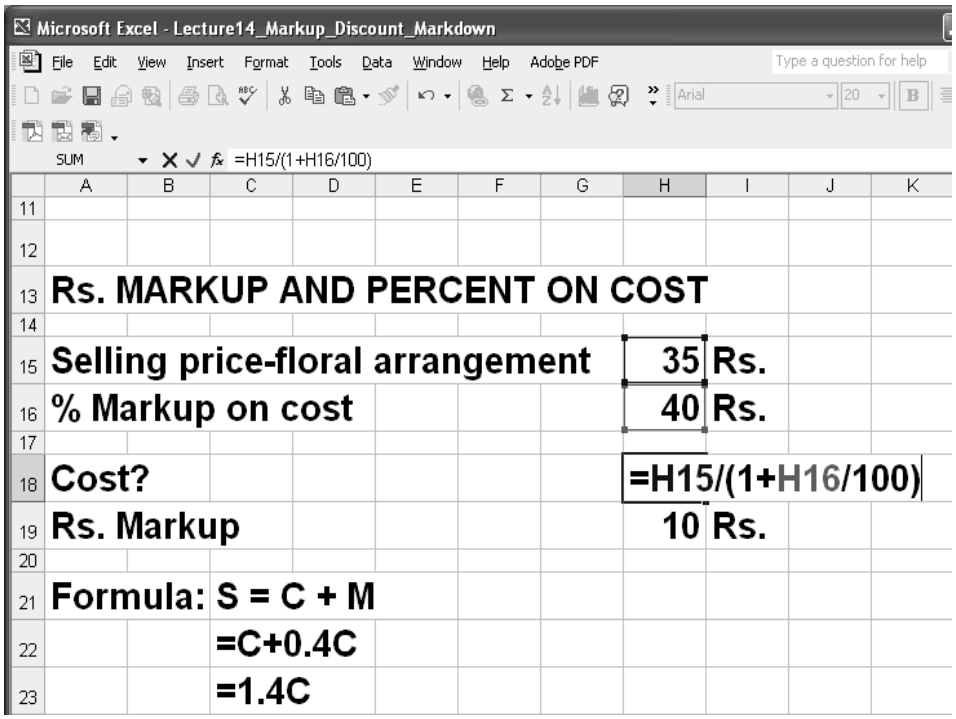
Tanveer's flower business sells floral arrangements for Rs. 35.
To make his desired profit, Tanveer needs a 40% Markup on cost.
What do the flower arrangements cost Tanveer?
What is the Rs. Markup?

Rs. Markup and Percent on Cost

Sale price S = Cost C + Markup M
S = C + .40(C)
35 = 1 .40(C)
C = 35/1,4 = 25 Rs.
M = 25 x 0.4
= 10 Rs.

EXCEL Calculation

Here 35 is the Selling price-floral arrangement in cell H15.
% Markup on cost is in cell H16.
EXCEL formula in cell H18 for Cost was based on the calculation = 35/1,4.
The Rs. Markup was calculated in cell H19 by using the formula =H18*H16/100.
The answer as shown in cell H19 was 10.



MARKUP AGAIN

You buy candles for 2 Rs.
You plan to sell them for 2.50 Rs
What is your Rs. Markup?
What is your Percent Markup on Selling Price?

Rs. Markup

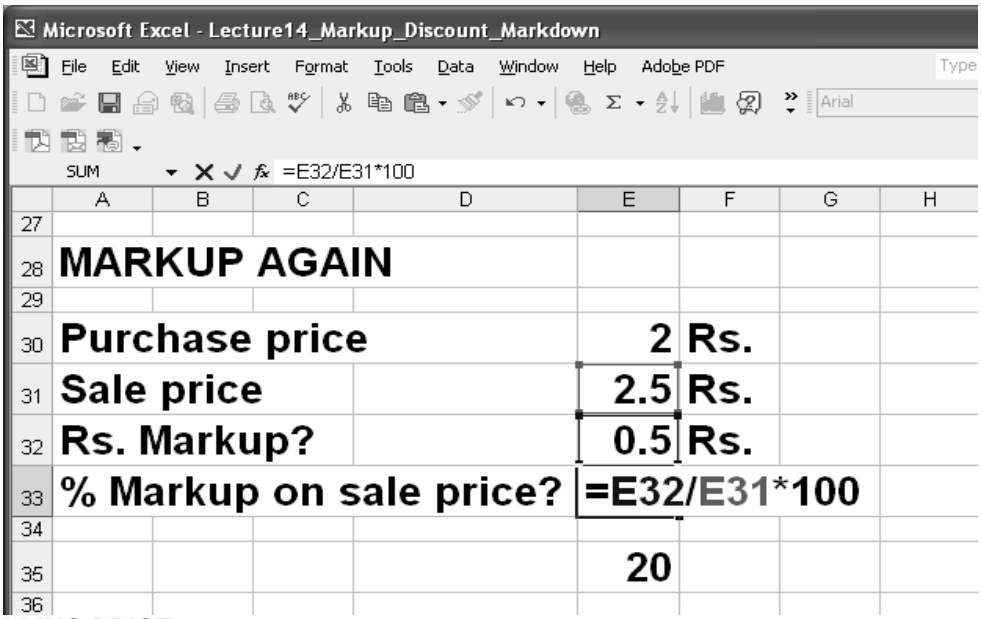
Rs. Markup = 2.5 – 2 = 0.5 Rs.

Percent Markup on Selling Price

Percent Markup on Selling Price = (0.5/2.5) x 100
= 20%

EXCEL Calculation

Here 2 is the Purchase price in cell E30.
Sale price is entered in cell E31.
Rs. Markup on Purchase Price was calculated by using the formula =E31-E30 in cell E32.
EXCEL formula in cell H18 for Cost was based on the calculation = 35/1,4.
The % Markup on sale price was calculated in cell E33 by using the formula =E32/E31*100.
The answer as shown in cell E35 was 10.



LLING PRICE

Fawad's Appliances bought a sewing machine for Rs. 1,500.
To make the desired profit, he needs a 60% Markup on Selling price.
What is Fawad's Rs. Markup?
What is his Selling Price?

Selling Price

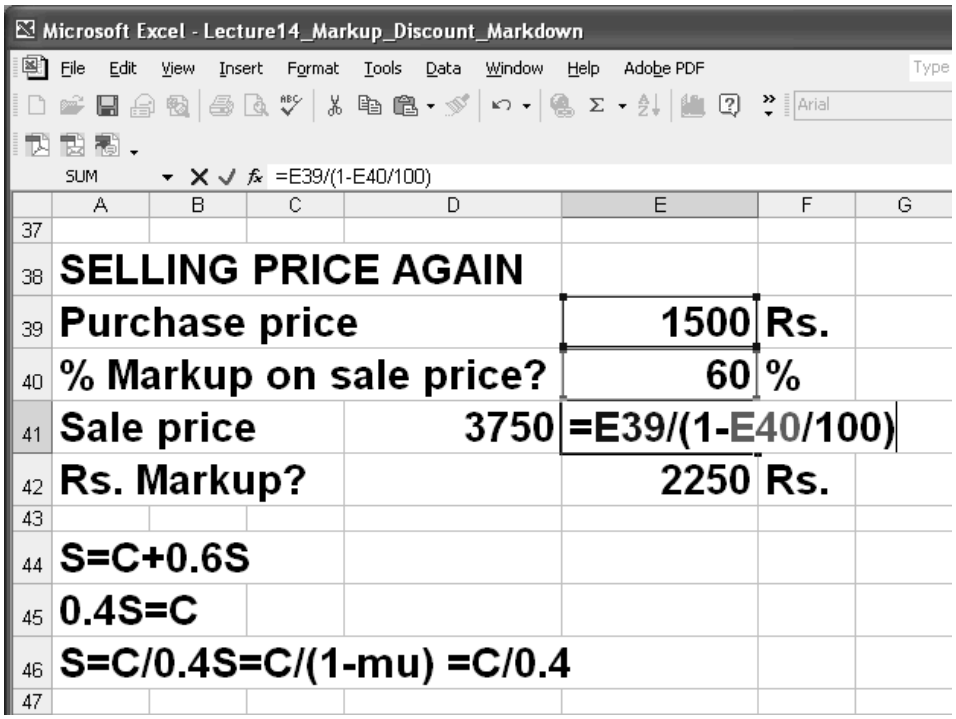
Selling Price S = 1,500 + 0.6S
S - 0.6S = 1,500 Rs.
Or
0.4S = 1,500 = 3,750 Rs

Rs. Markup

Rs. Markup = 3,750 x 0.6
= 2,250 Rs.

EXCEL Calculation

Here 1500 is the Purchase price in cell E39.
% Markup on Sale Price is entered as 60 in cell E40.
Sale Price was calculated by using the formula =E39/(1-E40/100). The result 3750 is shown in cell D41.
EXCEL formula in cell E42 for Rs. Markup was = E41-E39. The result 2250 is shown in cell E42.
Basic formula S=C+0.6S is shown in cell A44. In cell A45 it was simplified to 0.4=C. In cell A46, it is rewritten as S=C/0.4S=C/(1-mu)=C/0.4. Here mu is the Markup.



RS. MARKUP AND PERCENT ON COST

Tanveer's flower business sells floral arrangements for Rs. 35.
To make his desired profit, Tanveer needs a 40% Markup on Selling Price.
What do the flower arrangements Cost Tanveer?
What is the Rs. Markup?

Selling Price

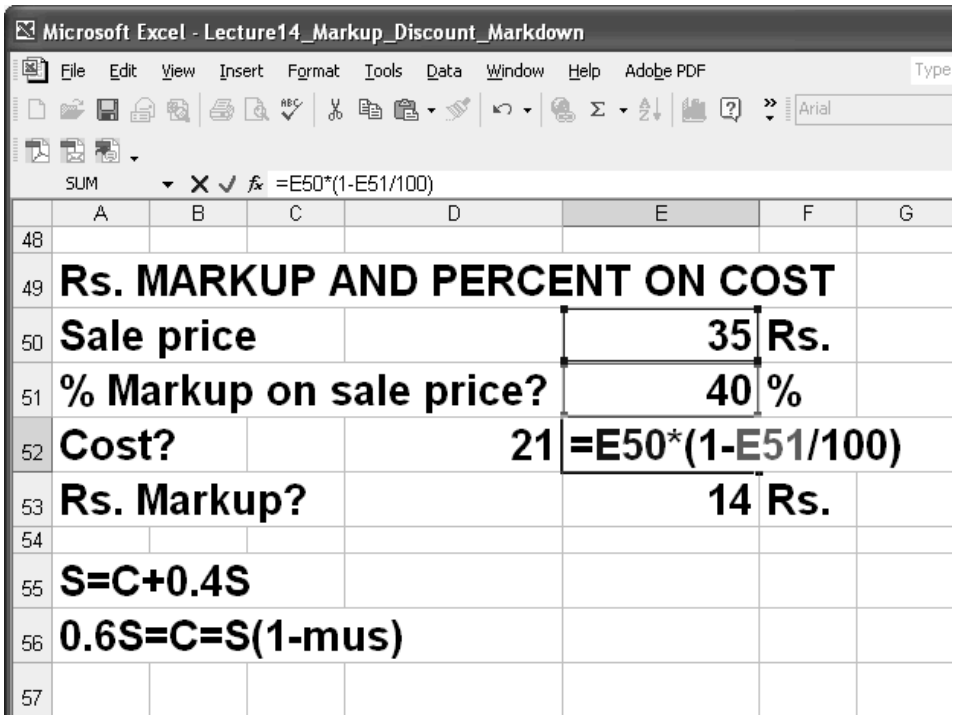
Selling Price =
35 = C + 0.4x 35
35 = C + 14
C = 35 – 14
= 21 Rs.
Or
C = S- 0.4 S
= 0.6 S = 0.6 x 35= 21 Rs.

Rs. Markup

Rs. Markup = 35 x 0.4 = 14 Rs.

EXCEL Calculation

Here 350 is the Sale price in cell E50.
% Markup on Sale Price is entered as 40 in cell E51.
Cost was calculated by using the formula =E50*(1-E51/100). The result 21 is shown in cell D52.
EXCEL formula in cell E53 for Rs. Markup was = E50-E52. The result 14 is shown in cell E53.
Basic formula S=C+0.4S is shown in cell A55. In cell A56 it was simplified to 0.6S=C=S(1-mus).



CONVERTING MARKUPS

Converting 50% Markup (MU) on Cost
= ? % MU on S

Formula

To convert % Markup on Selling Price (mus) to % Markup on Cost (muc):
% Markup on Selling Price (mus) as % of Cost =
% Markup on C/(1 + % Markup on C)
mus = muc/(1+muc)
= 0.5/(1+0.5) = 0.5/1.5
mus= 0.3333 = 33.33%

Converting Markups

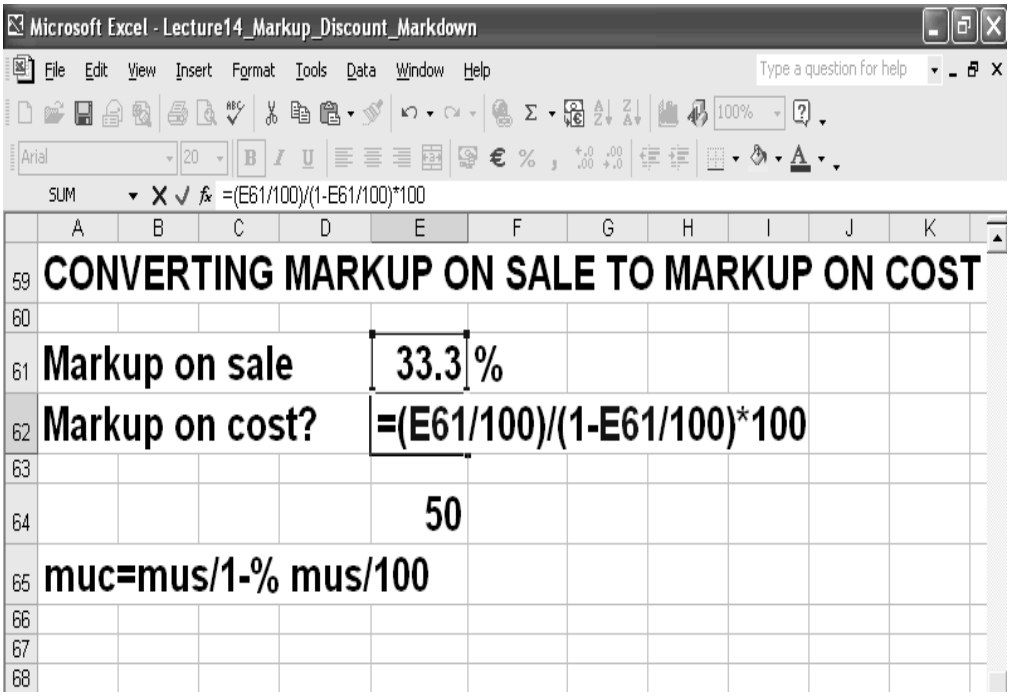
Converting 33.33% MU on Sale = ? % MU on C

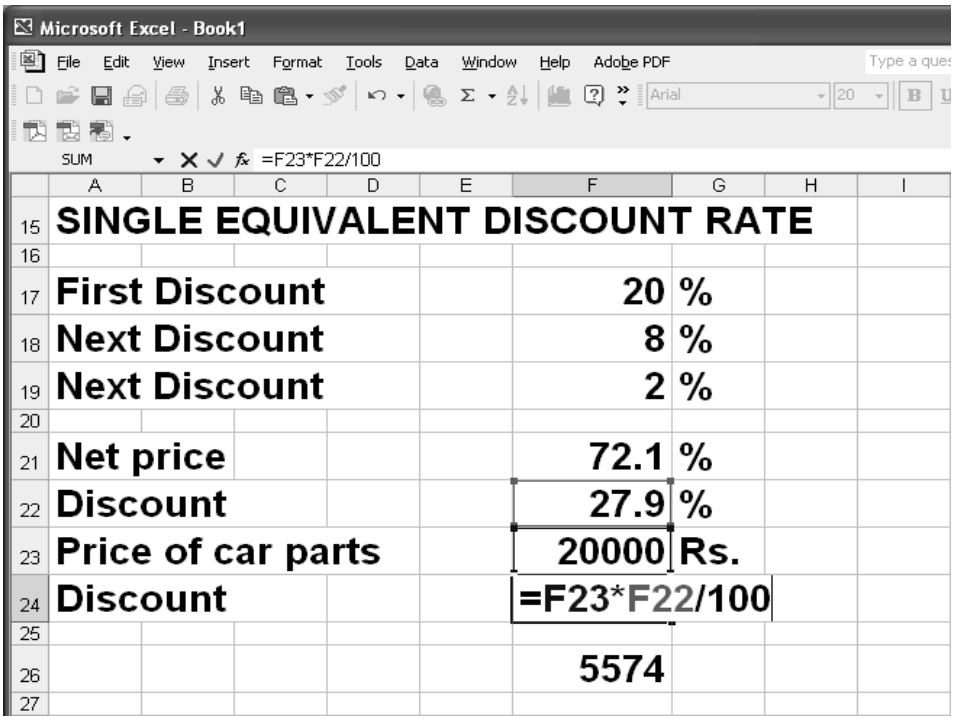
Formula

% Markup on Selling Price (mus) to % Markup on Cost (muc)=
% Markup on S/1 - % Markup on S
muc = mus/1-mus
= 0.3333/(1 – 0.333)
= 0.3333/0.6666 = 0.5
= 50%

EXCEL Calculation

Here 33.3 is the Markup on sale in cell E61.
EXCEL formula in cell E62 for Markup on cost was = (E61/100)/(1-E61/100)*100. The result 50 is shown in cell E64.
Basic formula muc=mus/(1-%mus/100) is shown in cell A65.





CASH DISCOUNT

A discount given for the prompt payment of an account is called Cash Discount. No Cash Discount is allowed on Invoices, Returned Goods, Freight, Sales Tax and Trade Discounts.

DISCOUNT PERIODS
Discount Periods are periods for the buyer to take advantage of Discount Terms.

CREDIT PERIODS
Credit Periods are periods for the buyers to pay invoices within specified times.

CASH DISCOUNT TERMS
Invoice was dated May 3. the Terms 2/10 mean that there is Discount 2% if invoice is paid upto 10 May. Then discount can be claimed

CASH DISCOUNT-EXAMPLE
What is the net payment for invoice value of Rs. 50,000 if paid upto 10 May?

Cash Discount
$N = L(1 – d)$
$= 50,000(1-0.02)$
$= 50,000(0.98)$
$= 49,000$ Rs.

EXCEL Calculation
EXCEL formula for net price was based on the calculation = 50,000(1-0.02). however, here an IF condition was applied, that means that if the payment date in cell D31 ($ sign is put in front of row and column to fix its location) is less than or equal to 10 May then the discount will be as given in cell d30. here also $ sign was used to fix the location of the cell.
In cell D38, the date was changed to 11 May and the same formula was applied again. The result as shown in cell D39 and D40 as 0% (% discount) and 0(Rs. Discount).

LECTURE 16
MATHEMATICS OF MERCHANDISING
PART 4

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 15
- Markup and Markdown

Financial Mathematics Part 3

MARKDOWN

Reduction from original selling Price is called Markdown.

Formula

%Markdown = (Rs. Markdown / Selling Price (original))*100

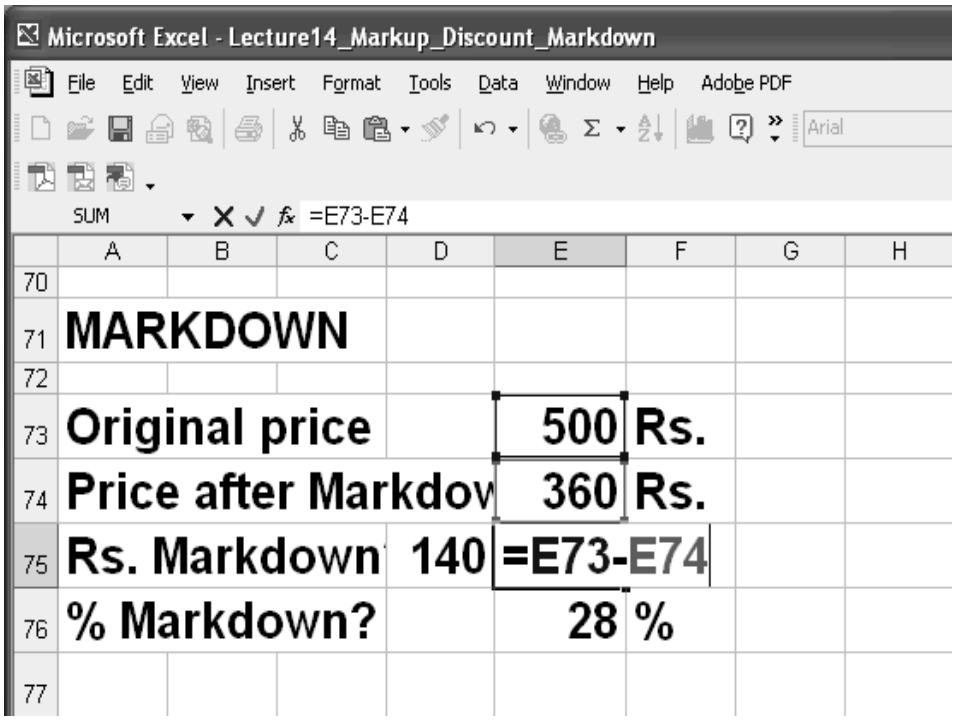
MARKDOWN-EXAMPLE 1

Store A marked down a Rs. 500 shirt to Rs. 360.
What is the Rs. Markdown?
What is the %markdown?
Rs. Markdown
Let S = Sale price
Rs. Markdown = Old S – New S
= Rs. 500 – Rs. 360
= Rs. 140 Markdown
% Markdown
% Markdown = Markdown *100
Old S
% Markdown = 140
500
= 0.28
= 28 %

EXCEL Calculation

Here 500 is the Original price in cell E73.
Price after Markdown is entered as 360 in cell E74.
Rs. Markdown was calculated in cell E75 by using the formula =E73-E74. The result 140 is shown in cell D75.

EXCEL formula in cell E76 for % Markdown was = E75/E73*100. The result 28 is shown



MARKDOWN-EXAMPLE 2

A variety of plastic jugs that was bought for Rs. 57.75, was marked up 45% of the SellingPrice.
When the jugs went out of production, they were marked down 40%
What was the Sale Price after the 40% markdown?
Here, there are two parts to this problem. First we must find the original price so that markdown can be calculated on that price.

Original Sale Price

Selling price = 100
Markup = 45
Cost = 100 – 45 = 55
Original Sale price = (100/55) x 57.75 = 105

Rs. Markdown

Markdown = 40 % = 0.4
Rs. Markdown = 105 x 0.4 = 42

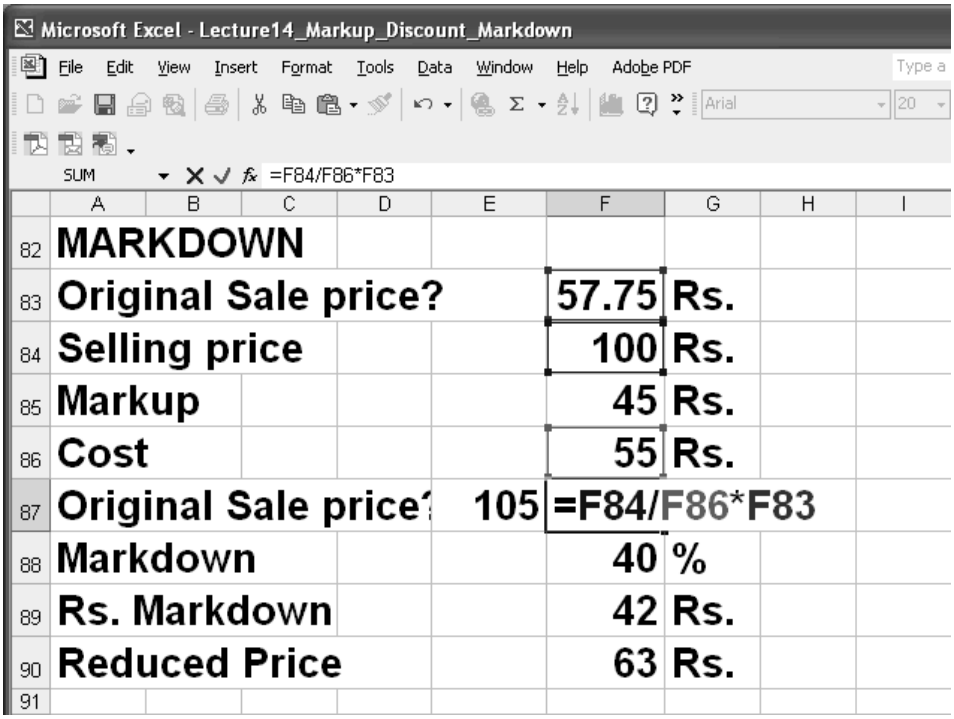
Sale price after markdown

Sale price after markdown = 105 – 42 = 63 Rs.

EXCEL Calculation

Here 57.75 is the Original sale price in cell F83.
Selling price is entered as 100in cell F84.
Rs. Markup was calculated in cell F85 using the formula =F84-F83. The result is shown as 45 in cell F85.
Original Sale Price was calculated in cell F87 by using the formula =F84/F86*F83. The result 105 is shown in cell E87.
% Markdown was entered as 40 in cell F88.
The Rs. Markdown was calculated using the formula =F87*F88/100 in cell F89. The result 42 is shown in cell F89.

The reduced price was calculated by using the formula =F87-F89 in cell F90. The result is shown as 63 in cell F90.



PROJECT FINANCIAL ANALYSIS

When you carry out Project Financial analysis, a number of Financial Calculations are required. The important ones are summarized below:

- Cost estimates
- Revenue estimates
- Forecasts of costs
- Forecasts of revenues
- Net cash flows
- Benefit cost analysis
- Internal Rate of Return
- Break-Even Analysis

COST ESTIMATES

In every project you will be required to prepare a cost estimate. Generally, such cost estimates cover calculations based on quantities and unit rates. Such calculations are done in the form of tabular worksheets. In large projects there may be a number of separate calculations for part projects. Such component costs are then combined to calculate total cost. These are simple worksheet calculations unless conditional processing is required. Such conditional processing is useful if unit prices are to be found for a specific model from a large database.

REVENUE ESTIMATES

Along with costs even revenues are calculated. These calculations are similar to component costs.

FORECASTS OF COSTS

Forecasting requires a technique for projections. Once such technique Time Series Analysis will be covered later in this course. Forecasting techniques vary from case to

case. The applicable method should be determined first. Calculation of future forecasts can then be done through worksheets.

FORECASTS OF REVENUES

These will be done similar to the forecast of costs. Here also the method must be determined first. Once the methodology is clear, then the worksheets can be prepared easily.

NET CASH FLOWS

The difference between Revenue and Cost is called the Net Cash flow. This is an important calculation as the entire Project Operation and Performance is based on its cash flows.

BENEFIT COST ANALYSIS

This is the end result of the Project Analysis. The ratio between Present Worth of Benefits and Costs is called the Benefit Cost (BC) ratio. For a project to be viable without profit or loss, the BC Ratio must be 1 or more. Generally a BC Ratio of 1.2 is considered acceptable. For Public projects even lesser BC ratio may be accepted for social reasons.

INTERNAL RATE OF RETURN

Internal Rate of Return or IRR is that Discount Rate at which the Present Worth of Costs is equal to the Present Worth of Benefits. IRR is the most important parameter in Financial and Economic Analysis. There are a number of functions in EXCEL for calculation of IRR.

BREAK-EVEN ANALYSIS

In every project where investment is made it is important to know how long it takes to recover the investment. It is also important to find the breakeven point where the Cash Inflow becomes equal to Cash Outflow. After that point the company has a positive cash flow (i.e. there is surplus cash after meeting expenses).

LECTURE 17
MATHEMATICS FINANCIAL MATHEMATICS
INTRODUCTION TO SIMULTANEOUS EQUATIONS

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 16
- Financial Mathematics
- Introduction to Linear Equations

MARKDOWN

Reduction from original selling Price is called Markdown.

Module 4

Module 4 covers the following:

- Financial Mathematics (Lecture 17)
- Applications of Linear Equations
- (Lecture 17-18)
- Break-even Analysis
- (Lectures 19-22)
- Mid-Term Examination

PROJECT FINANCIAL ANALYSIS

Project Financial Analysis covers the following:

- Cost estimates
- Revenue estimates
- Forecasts of costs
- Forecasts of revenues
- Net cash flows
- Benefit cost analysis
- Internal Rate of Return
- Break-Even Analysis

EXCEL FUNCTIONS FINANCIAL ANALYSIS

List of Excel Financial functions is as under. The name and utility of each function is given below:

AMORDEGRC Returns the depreciation for each accounting period by using a depreciation coefficient

AMORLINC

Returns the depreciation for each accounting period.

AMORDEGRC

Depreciation Using Depreciation Coefficient

AMORDEGRC(cost,date_purchased,first_period,salvage,period,rate,basis)

Cost cost of the asset.

Date_purchased date of the purchase of the asset.

First_period date of the end of the first period.

Salvage salvage value at the end of the life of the asset.

Period period.

Rate rate of depreciation.

Basis year basis to be used.

AMORLINC

Depreciation Using Prorated Depreciation

AMORLINC(cost,date_purchased,first_period,salvage,period,rate,basis)

Cost cost of the asset

Date_purchased date of the purchase of the asset.

First_period date of the end of the first period.

Salvage salvage value at the end of the life of the asset.

Period period.

Rate rate of depreciation.

Basis year basis to be used.

AMORLINC-EXAMPLE

Data	**Description**
2400	Cost
2401	8/19/2008 Date purchased
2402	12/31/2008 End of the first period
2403	300 Salvage value
2404	1 Period
2405	15% Depreciation rate
	Actual basis (see above)

(Result)=AMORLINC(A2,A3,A4,A5,A6,A7,A7)First period depreciation (360)

CUMIPMT

Returns the cumulative interest paid between two periods

CUMPRINC

Returns the cumulative principal paid on a loan between two periods

DB

Returns the depreciation of an asset for a specified period using the fixed-declining balance method

DDB

Returns the depreciation of an asset for a specified period using the double-declining balance method or some other method you specify

IRR

Returns the internal rate of return for a series of cash flows

MIRR

Returns the internal rate of return where positive and negative cash flows are financed at different rates

INTERNAL RATE OF RETURN IRR

IRR(values,guess)

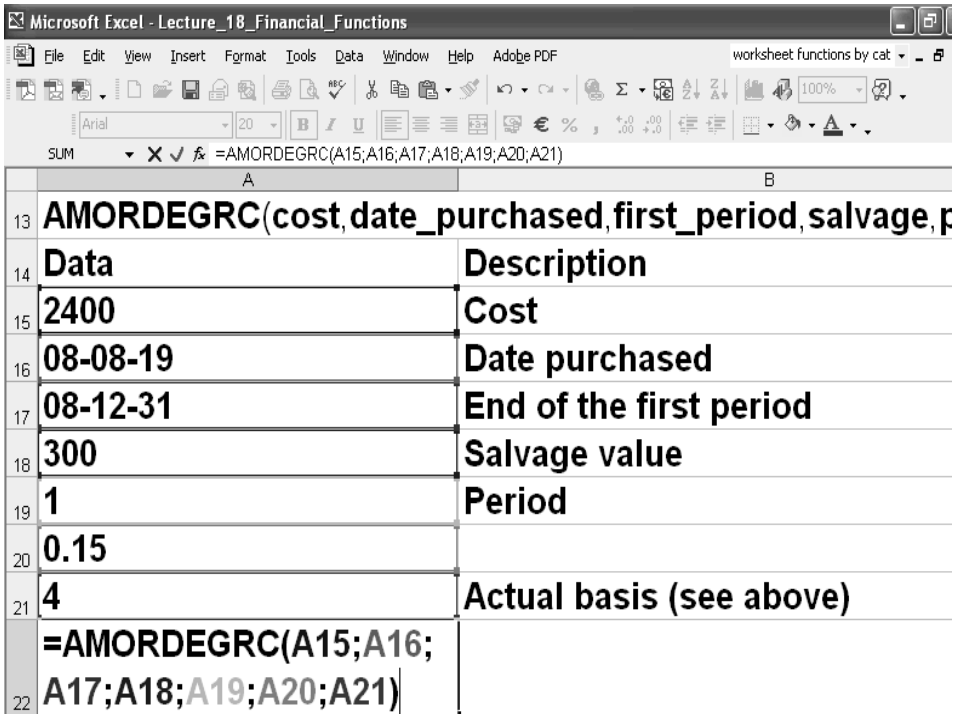
LECTURE 18
MATHEMATICS FINANCIAL MATHEMATICS
SOLVE TWO LINEAR EQUATIONS WITH TWO UNKNOWNS

OBJECTIVES

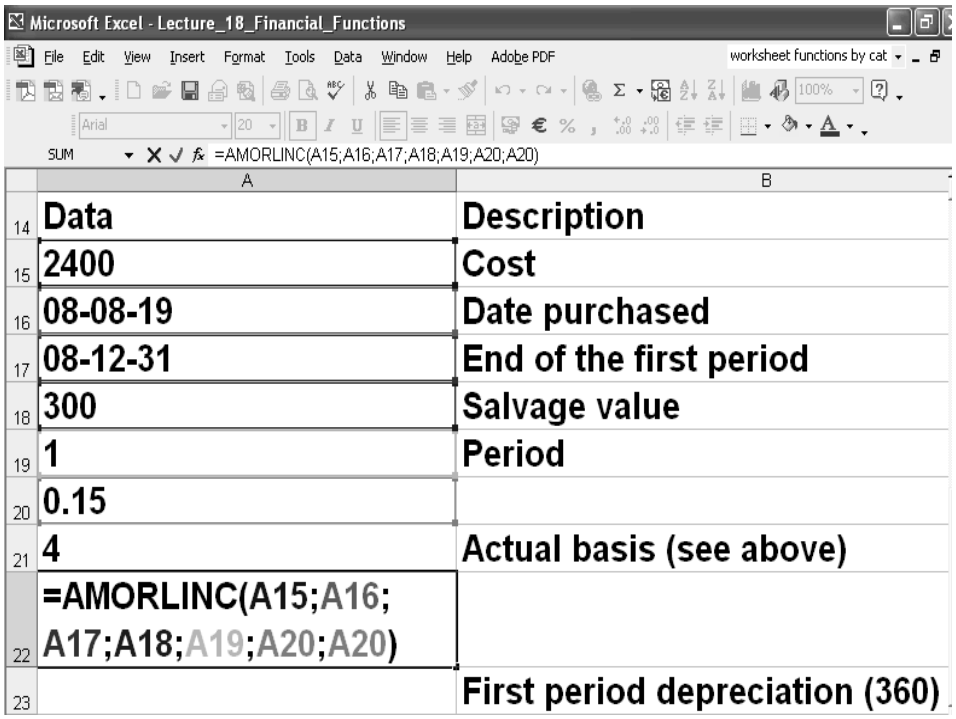
The objectives of the lecture are to learn about:

- Review Lecture 17
- Solve two linear equations with two unknowns

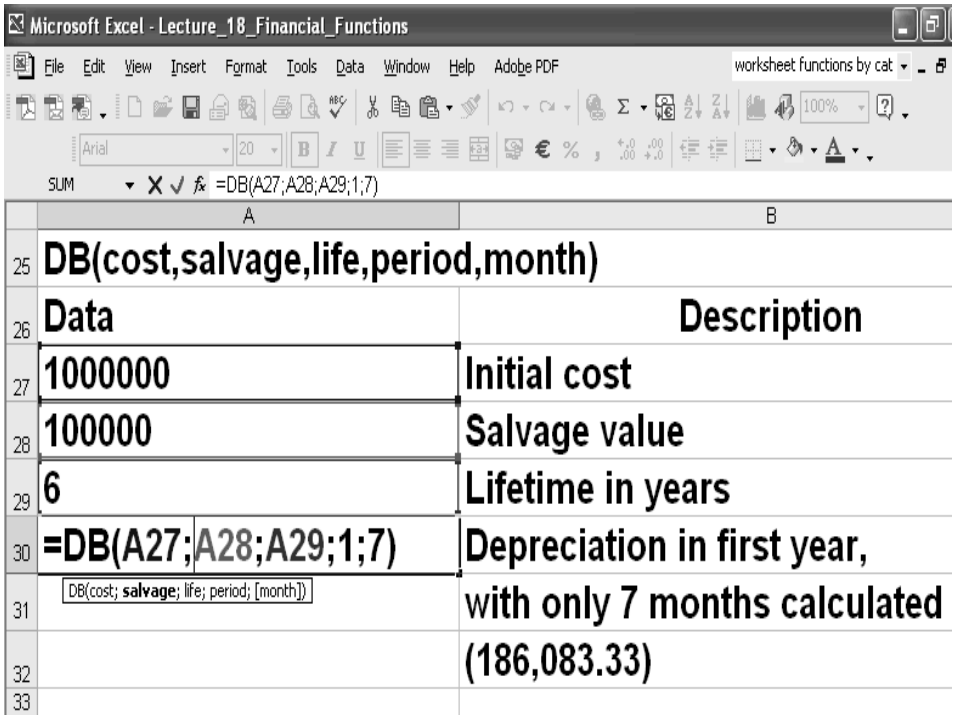
AMORDEGRC-EXAMPLE



AMORLINC-EXAMPLE



DB-EXAMPLE



ADDITIONAL DB_EXAMPLES

Look at the following examples to see how the DB function can be used in different ways.

=DB(A2,A3,A4,1,7) Depreciation in first year, with only 7 months calculated (186,083.33)

- =DB(A2,A3,A4,2,7) Depreciation in second year (259,639.42)

•=DB(A2,A3,A4,3,7) Depreciation in third year (176,814.44)
•=DB(A2,A3,A4,4,7) Depreciation in fourth year (120,410.64)
•=DB(A2,A3,A4,5,7) Depreciation in fifth year (81,999.64)
•=DB(A2,A3,A4,6,7) Depreciation in sixth year (55,841.76)
•=DB(A2,A3,A4,7,5) Depreciation in seventh year, with only 5 months calculated (15,845.10)

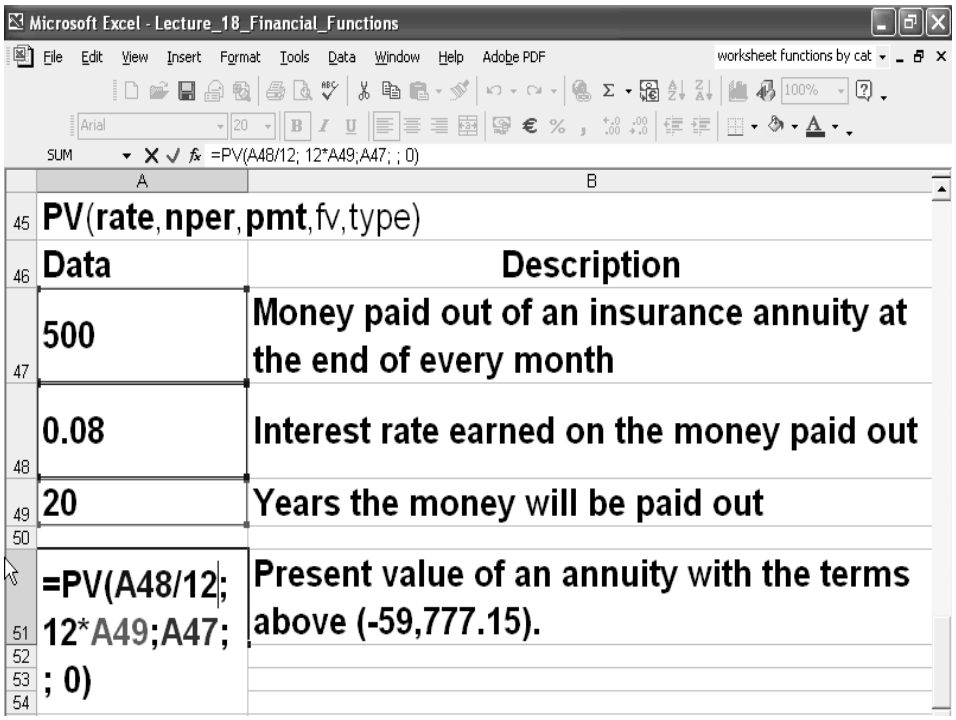
PV

Returns the present value of an investment

PV(rate,nper,pmt,fv,type)

Rate interest rate per period

Nper total number of payment periods in an annuity
Pmt payment made each period and cannot change over the life of the annuity
Fv future value, or a cash balance you want to attain after the last payment is made
Type number 0 or 1 and indicates when payments are due.



NPV

Returns the net present value of an investment based on a series of periodic cash flows and a discount rate

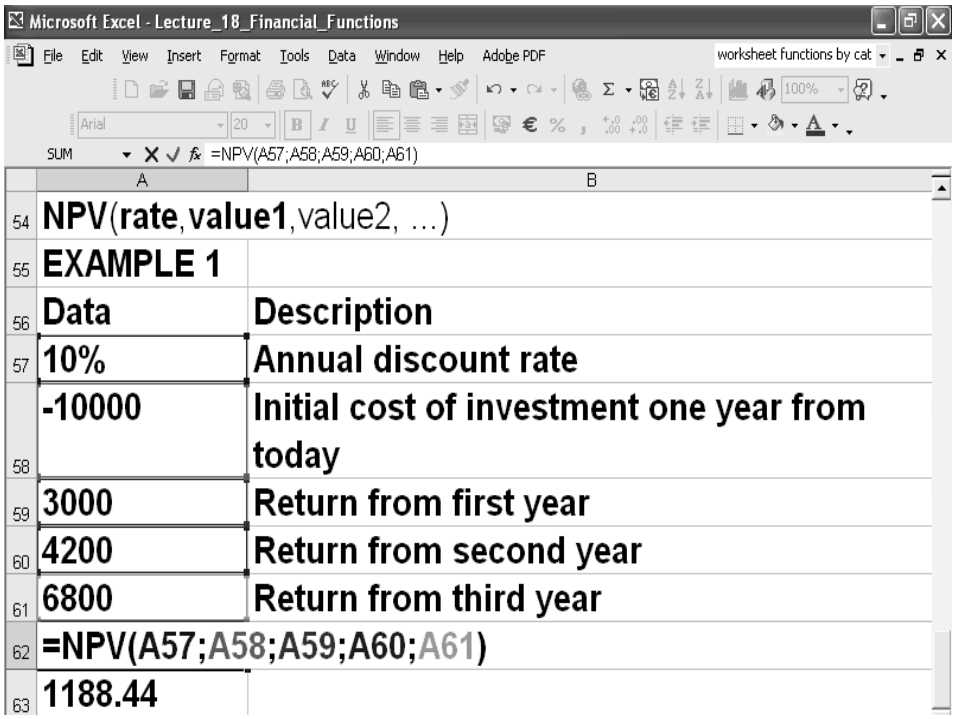
NPV(rate,value1,value2, ...)

Rate

rate of discount over the length of one period

Value1, value2, ...

1 to 29 arguments representing the payments and income



XNPV

Returns the net present value for a schedule of cash flows that is not necessarily periodic **XNPV(rate,values,dates)**

Rate

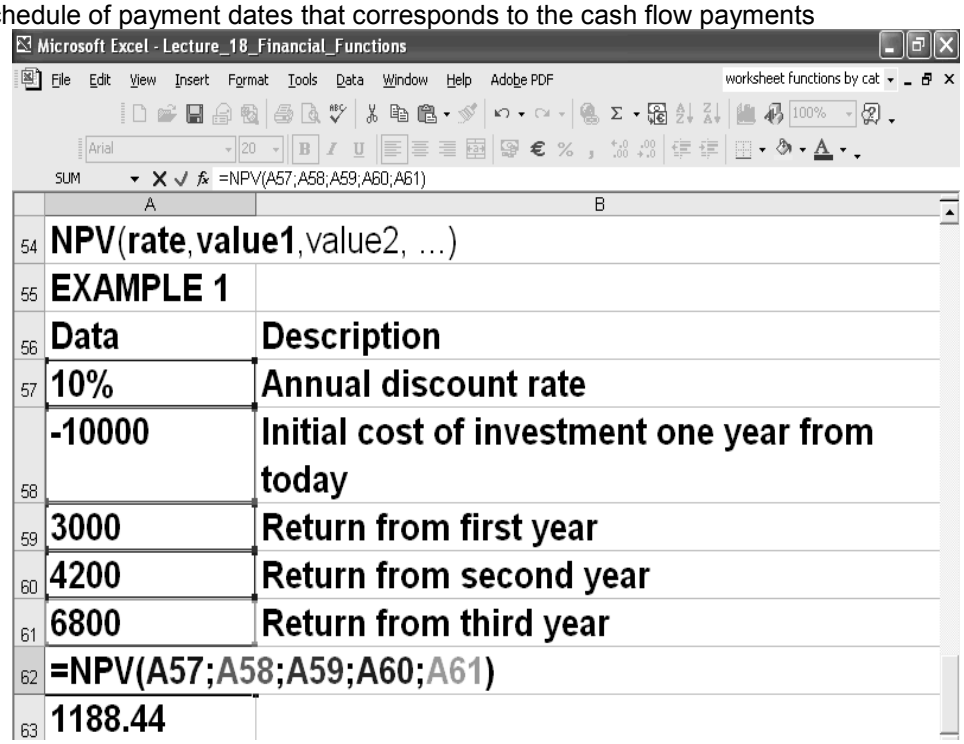
discount rate to apply to the cash flows

Values

series of cash flows that corresponds to a schedule of payments in dates

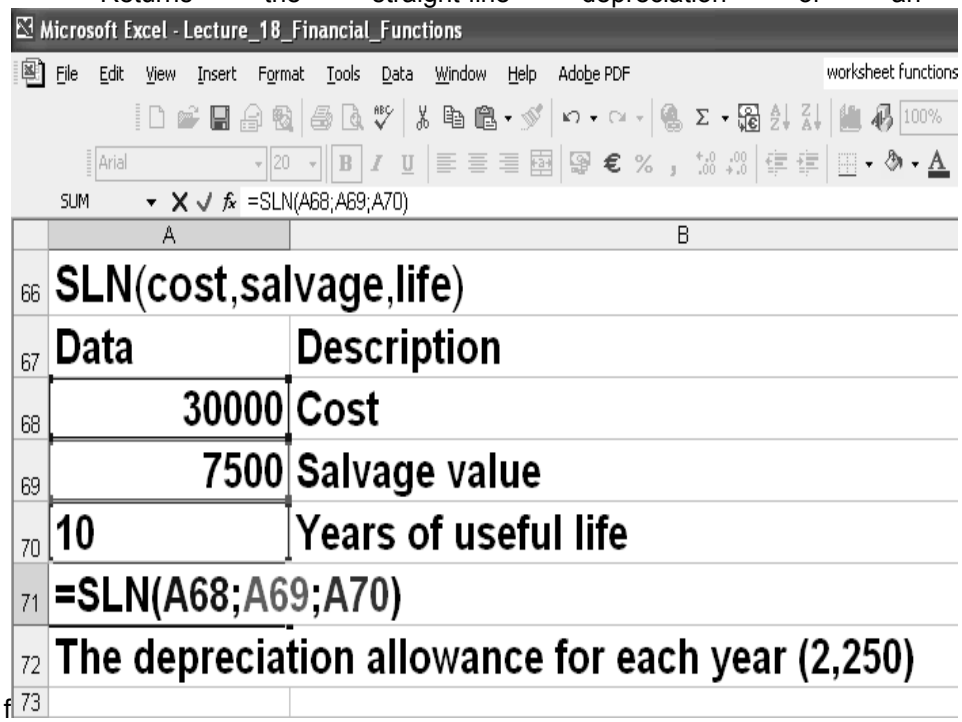
Dates

schedule of payment dates that corresponds to the cash flow payments



SLN

Returns the straight-line depreciation of an asset



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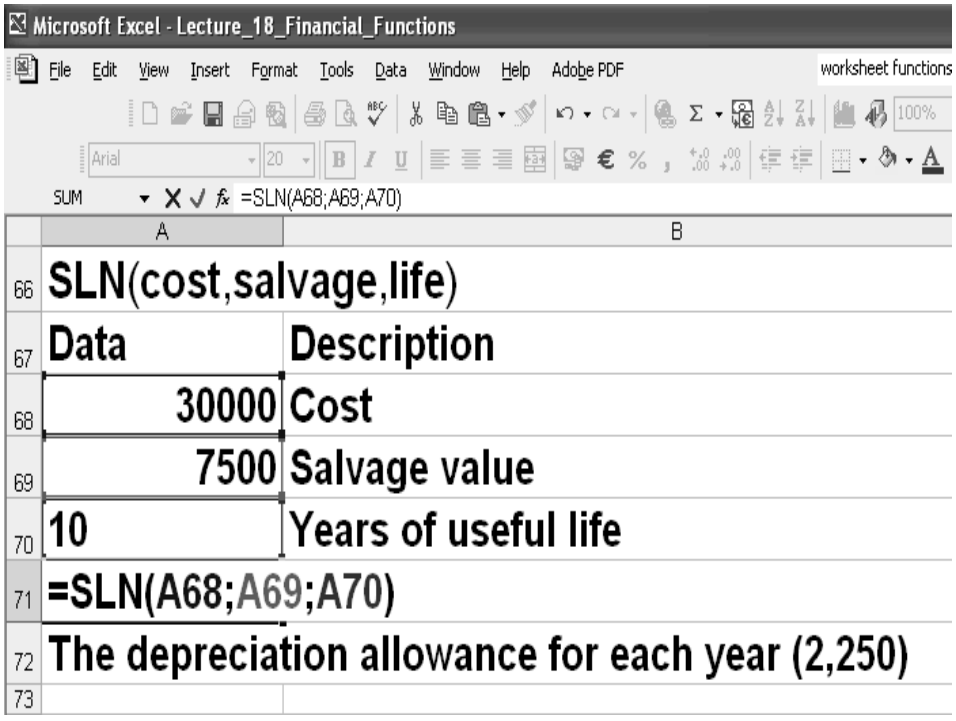
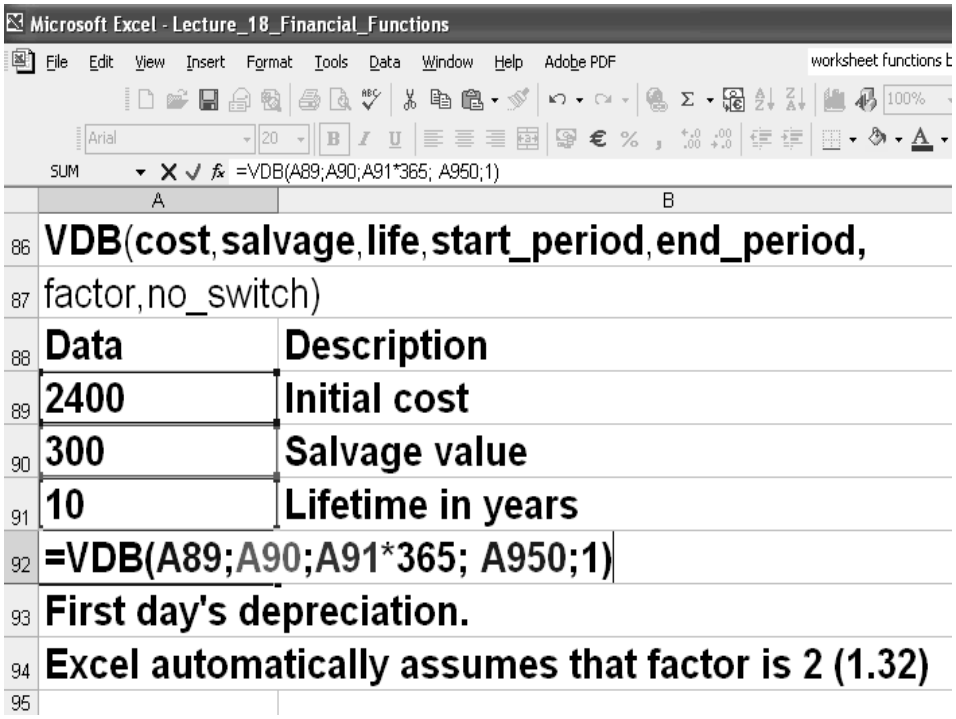
SYD

Returns the sum-of-years' digits depreciation of an asset for a specified period



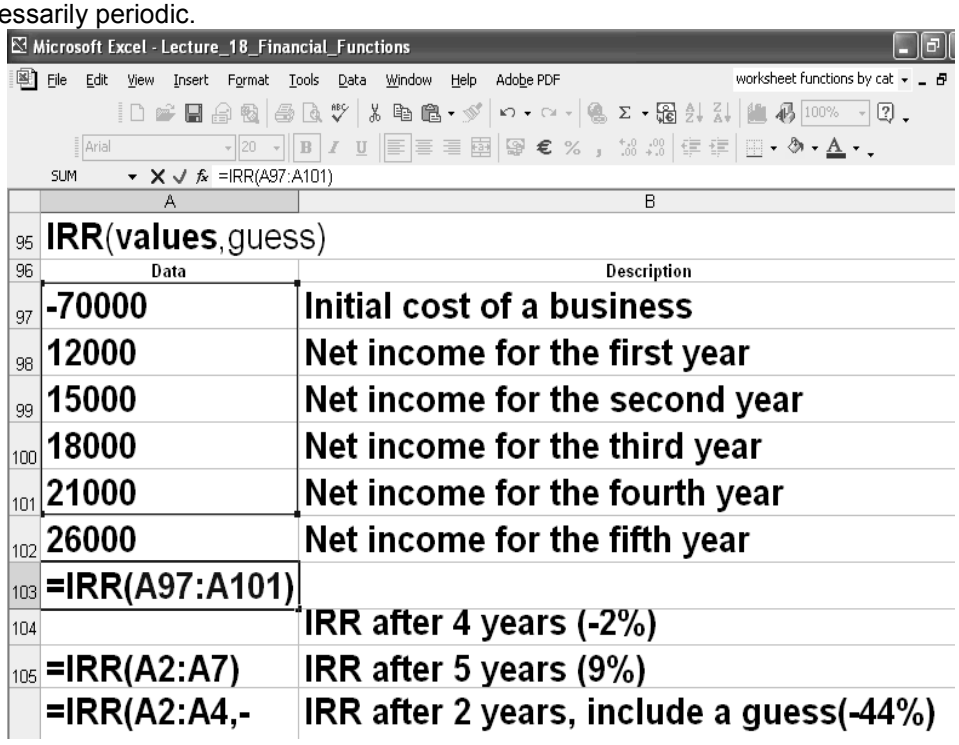
VDB

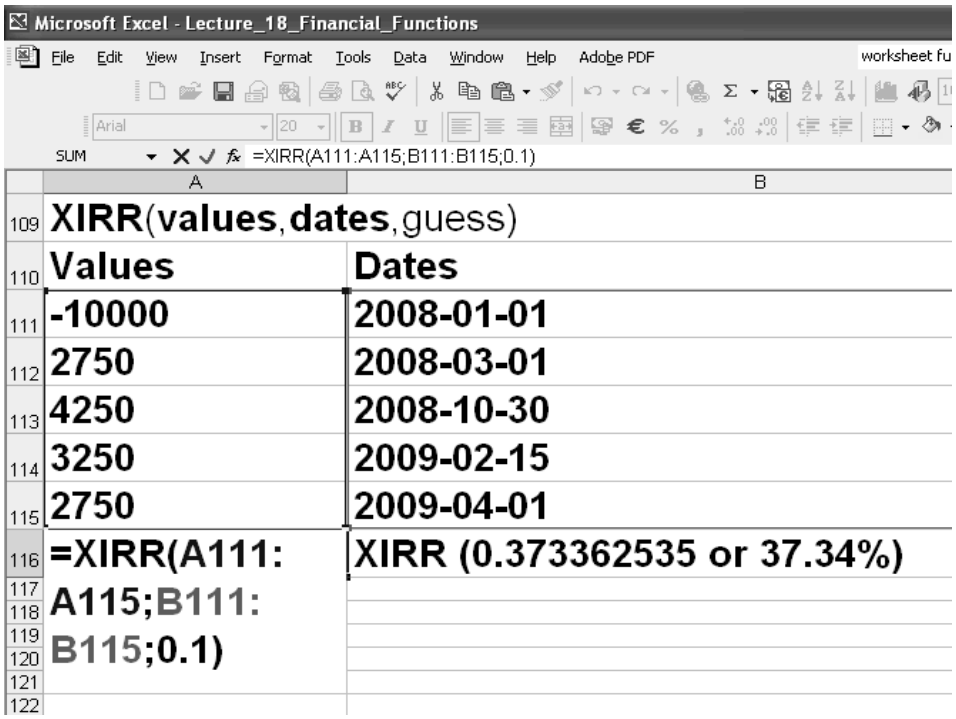
Returns the depreciation of an asset for a specified or partial period using a declining balance method



XIRR

Returns the internal rate of return for a schedule of cash flows that is not necessarily periodic.





LINEAR EQUATIONS

- Solve two linear equations with two variables
- Solve problems that require setting up linear equations with two variables
- Perform linear Cost-Volume-Profit and break-even analysis employing:
- The contribution margin approach
- The algebraic approach of solving the cost and revenue functions

SOLVING LINEAR EQUATIONS-PART 1

Here is an example of solving simultaneous linear equations.

$2x - 3y = -6$

$x + y = 2$

Solve for y

$2x - 3y = -6$

$2x + 2y = 4$

$-5y = -10$

$y = 10/5$

$y = 2$

SOLVING LINEAR EQUATIONS - PART 2

Let us look at the same equations again.

$2x - 3y = -6$

$x + y = 2$

We solved for x.

Now let us substitute y by 2

$2x - 3(2) = -6$

$2x - 6 = -6$

$2x = 0$

$x = 0$

LECTURE 19
PERFORM BREAK-EVEN ANALYSIS
EXCEL FUNCTIONS FINANCIAL ANALYSIS

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 18
- Perform break-even analysis
- MS EXCEL Financial Functions

SLN-EXAMPLE

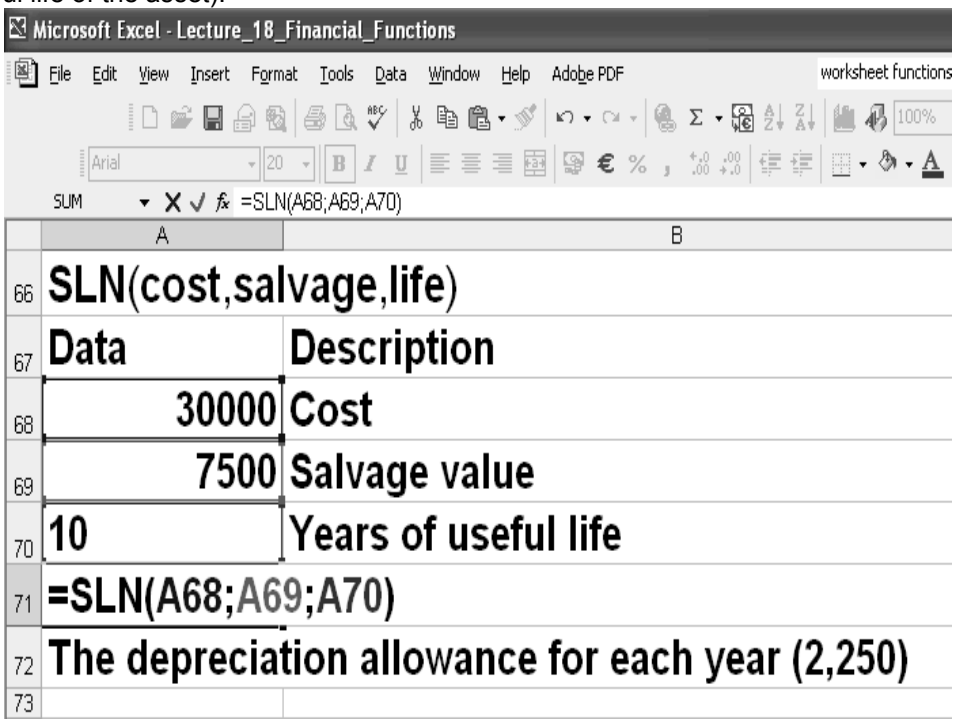
SLN ***Returns the straight-line depreciation of an asset for one period***

Syntax: SLN(cost,salvage,life)

Cost is the initial cost of the asset.

Salvage is the value at the end of the depreciation (sometimes called the salvage value of the asset).

Life is the number of periods over which the asset is depreciated (sometimes called the useful life of the asset).



SYD

Returns the sum-of-years' digits depreciation of an asset for a specified period

Syntax

SYD(**cost**,**salvage**,**life**,**per**)

Cost is the initial cost of the asset.

Salvage is the value at the end of the depreciation (sometimes called the salvage value of the asset).

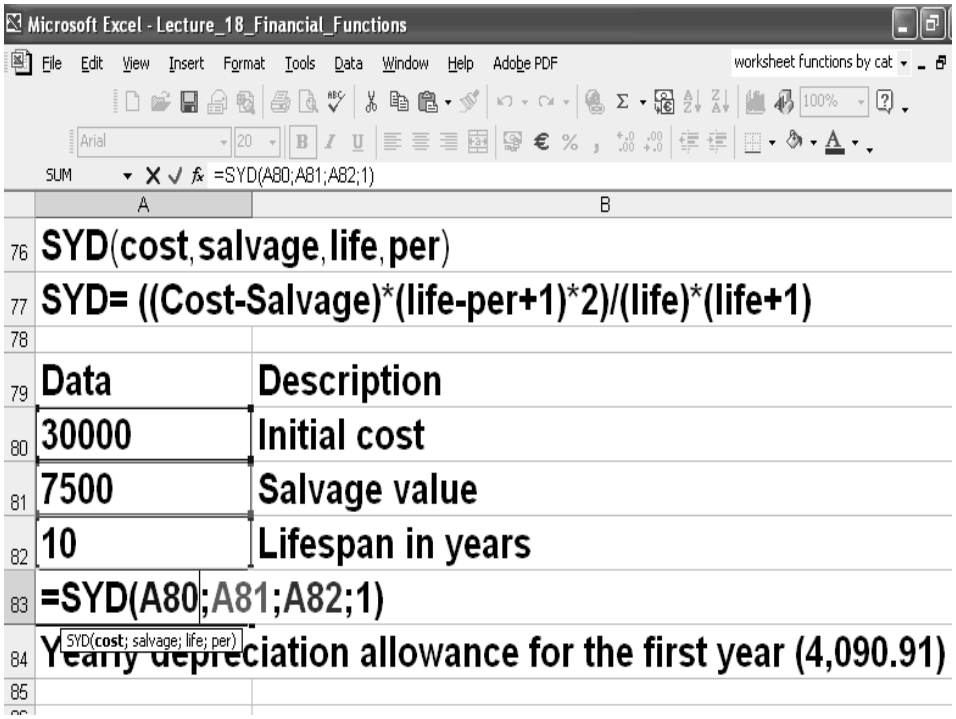
Life is the number of periods over which the asset is depreciated (sometimes called the useful life of the asset).

Per is the period and must use the same units as life.

Remark

- SYD is calculated as follows:

$$SYD = \frac{(cost - salvage) * (life - per + 1) * 2}{(life)(life + 1)}$$



VDB

Returns the depreciation of an asset for any period you specify, including partial periods, using the double-declining balance method or some other method you specify. VDB stands for variable declining balance.

Syntax

VDB(cost,salvage,life,start_period,end_period,factor,no_switch)

Cost is the initial cost of the asset.

Salvage is the value at the end of the depreciation (sometimes called the salvage value of the asset).

Life is the number of periods over which the asset is depreciated (sometimes called the useful life of the asset).

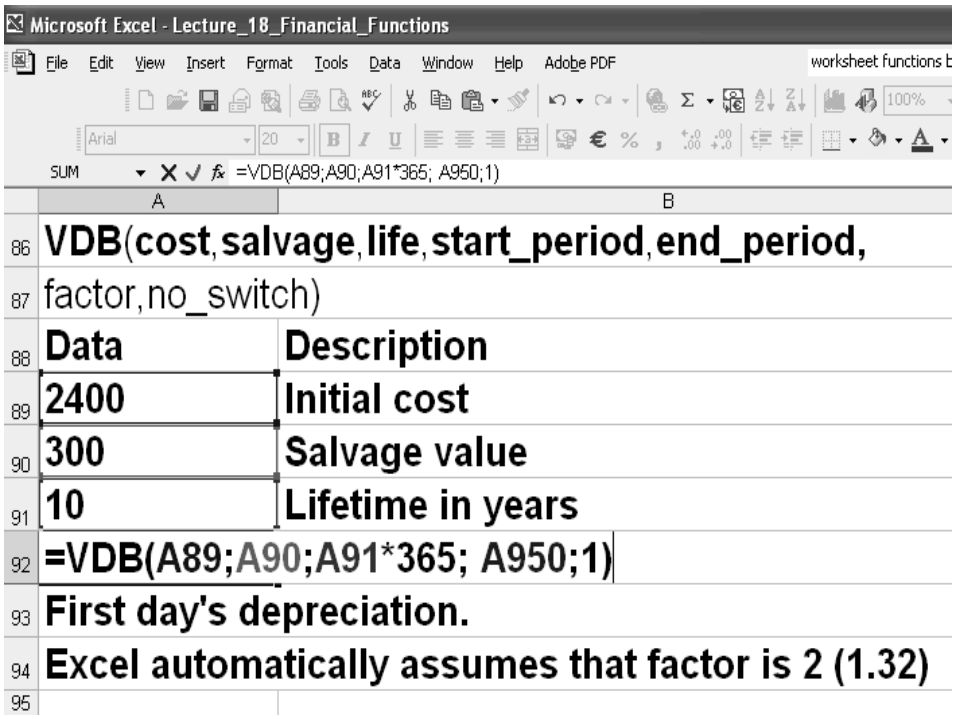
Start_period is the starting period for which you want to calculate the depreciation. Start_period must use the same units as life.

End_period is the ending period for which you want to calculate the depreciation. End_period must use the same units as life.

Factor is the rate at which the balance declines. If factor is omitted, it is assumed to be 2 (the double-declining balance method). Change factor if you do not want to use the double-declining balance method. For a description of the double-declining balance method, see DDB.

No_switch is a logical value specifying whether to switch to straight-line depreciation when depreciation is greater than the declining balance calculation.

- If **no_switch** is **TRUE**, Microsoft Excel does not switch to straight-line depreciation even when the depreciation is greater than the declining balance calculation.
- If **no_switch** is **FALSE** or omitted, Excel switches to straight-line depreciation when depreciation is greater than the declining balance calculation.
  All arguments except no_switch must be positive numbers.



IRR

Returns the internal rate of return for a series of cash flows represented by the numbers in values. These cash flows do not have to be even, as they would be for an annuity. However, the cash flows must occur at regular intervals, such as monthly or annually. The internal rate of return is the interest rate received for an investment consisting of payments (negative values) and income (positive values) that occur at regular periods.

Syntax

IRR(**values**,guess)

Values is an array or a reference to cells that contain numbers for which you want to calculate the internal rate of return.

- **Values** must contain at least one positive value and one negative value to calculate the internal rate of return.
- **IRR** uses the order of values to interpret the order of cash flows. Be sure to enter your payment and income values in the sequence you want.
- If an array or reference argument contains text, logical values, or empty cells, those values are ignored.

  Guess is a number that you guess is close to the result of IRR.
- Microsoft Excel uses an iterative technique for calculating IRR. Starting with guess, IRR cycles through the calculation until the result is accurate within 0.00001 percent. If IRR can't find a result that works after 20 tries, the #NUM! error value is returned.
- In most cases you do not need to provide guess for the IRR calculation. If guess is omitted, it is assumed to be 0.1 (10 percent).
- If IRR gives the #NUM! error value, or if the result is not close to what you expected, try again with a different value for guess.

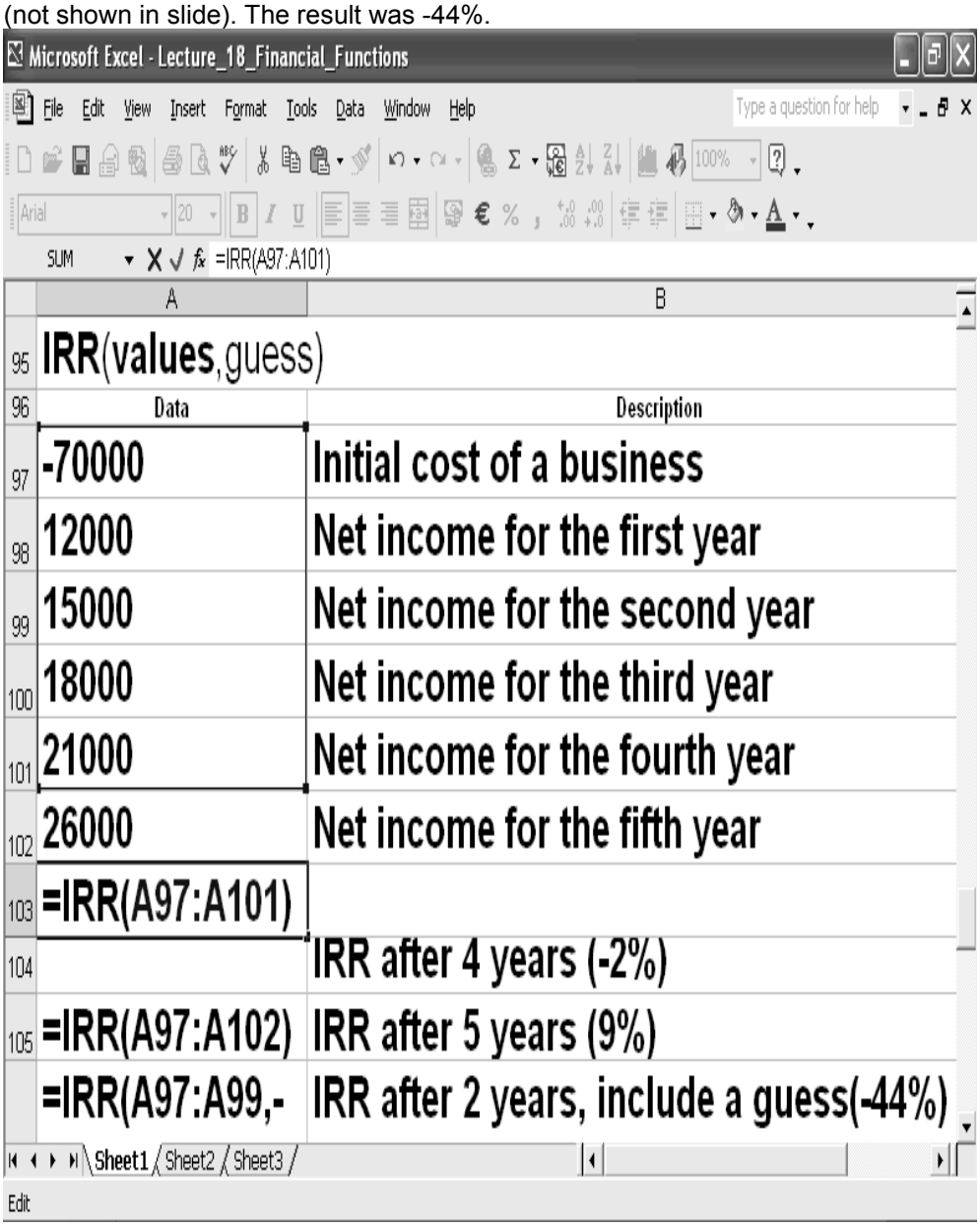
Remarks

IRR is closely related to **NPV**, the net present value function. The rate of return calculated by IRR is the interest rate corresponding to a 0 (zero) net present value. The following formula demonstrates how NPV and IRR are related:

NPV(IRR(B1:B6),B1:B6) equals 3.60E-08 [Within the accuracy of the IRR calculation, the value 3.60E-08 is effectively 0 (zero).]

IRR-EXAMPLE

In the slide the Excel worksheet is shown.
In cell A97, the investment of 70,000 is entered with minus sign to denote negative cash flow.
In cell A98 to A102, revenue per year (1 to 5) is entered.
In the first formula in cell A103 (=IRR(A97:A101)), only years 1 to 4 were selected for the revenue stream. The IIR is -2% in this case.
In the next formula in cell A105, the entire revenue stream was considered. The IRR improved to 9%.
Next only first 2 years of revenue stream were considered with an initial guess of 10% (not shown in slide). The result was -44%.



XIRR

Returns the internal rate of return for a schedule of cash flows that is not necessarily periodic. To calculate the internal rate of return for a series of periodic cash flows, use the IRR function.

If this function is not available, and returns the #NAME? error, install and load the Analysis ToolPak add-in. To do that:

1. On the **Tools** menu, click **Add-Ins**.
2. In the **Add-Ins available** list, select the **Analysis ToolPak** box, and then click **OK**.
3. If necessary, follow the instructions in the setup program.

Syntax

XIRR(**values**,**dates**,**guess**)

Values is a series of cash flows that corresponds to a schedule of payments in dates. The first payment is optional and corresponds to a cost or payment that occurs at the beginning of the investment. If the first value is a cost or payment, it must be a negative value. All succeeding payments are discounted based on a 365-day year. The series of values must contain at least one positive and one negative value.

Dates is a schedule of payment dates that corresponds to the cash flow payments. The first payment date indicates the beginning of the schedule of payments. All other dates must be later than this date, but they may occur in any order. Dates should be entered by using the DATE function, or as results of other formulas or functions. For example, use DATE(2008,5,23) for the 23rd day of May, 2008. Problems can occur if dates are entered as text.

Guess is a number that you guess is close to the result of XIRR.

Remarks

- Microsoft Excel stores dates as sequential serial numbers so they can be used in calculations. By default, January 1, 1900 is serial number 1, and January 1, 2008 is serial number 39448 because it is 39,448 days after January 1, 1900. Microsoft Excel for the Macintosh uses a different date system as its default.
- Numbers in dates are truncated to integers.
- XIRR expects at least one positive cash flow and one negative cash flow; otherwise, XIRR returns the #NUM! error value.
- If any number in dates is not a valid date, XIRR returns the #VALUE! error value.
- If any number in dates precedes the starting date, XIRR returns the #NUM! error value.
- If values and dates contain a different number of values, XIRR returns the #NUM! error value.
- In most cases you do not need to provide guess for the XIRR calculation. If omitted, guess is assumed to be 0.1 (10 percent).
- XIRR is closely related to XNPV, the net present value function. The rate of return calculated by XIRR is the interest rate corresponding to XNPV = 0.
- Excel uses an iterative technique for calculating XIRR. Using a changing rate (starting with guess), XIRR cycles through the calculation until the result is accurate within 0.000001 percent. If XIRR can't find a result that works after 100 tries, the #NUM! error value is returned. The rate is changed until:

where:

$$0 = \sum_{i=1}^{N} \frac{P_i}{(1+rate)^{\frac{(d_i - d_1)}{365}}}$$

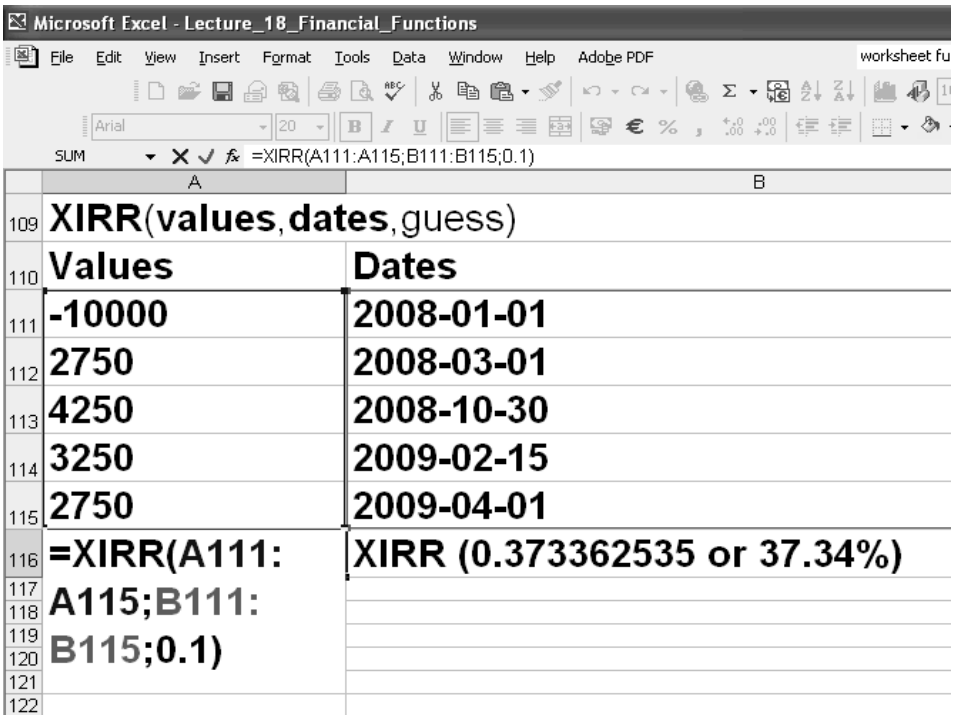
di = the ith, or last, payment date.

d1 = the 0th payment date.

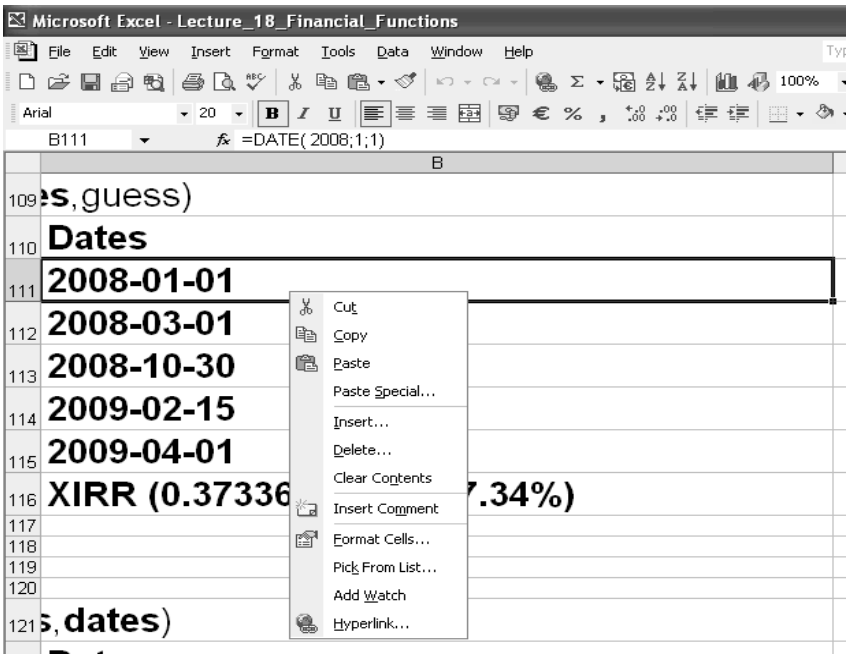
Pi = the ith, or last, payment.

XIRR EXAMPLE

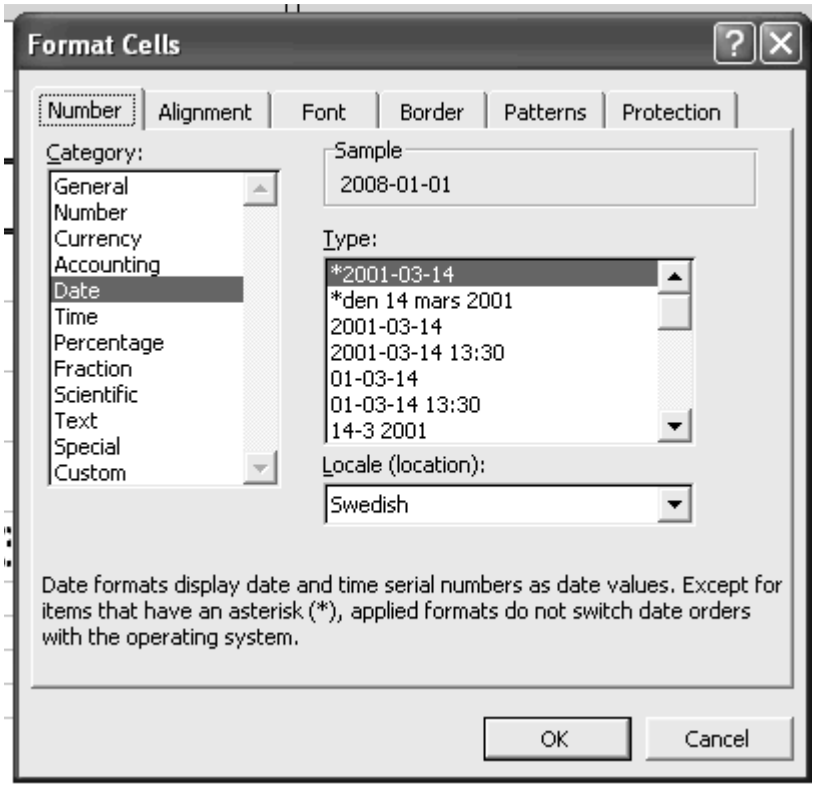
Here, the investment is in cell A111. The revenue stream is in cells A112 to a115. The dates for each investment or revenue are given in cells B111 to B115. Please note that the dates are in European format year-month-day. On your computer, you may not have this format.



After entering these days in Excel, you can right click on the cell. You see a short cut menu as shown below.



When you will select **Format Cells**, the Format Cells Dialog Box appears as shown below. You can then choose the desired format for the date.



In cell A116, the formula (XIIR(A111:A115;B111:B115;0.1), the range A111:A115 is the cost and revenue stream. The range B111:B115 is the stream for dates. The third term 0.1 is the initial guess for XIRR.

The answer in fraction or % is given in cell B116(37.34%).

LINEAR EQUATIONS

Linear equations have following applications in Merchandising Mathematics:

- Solve two linear equations with two variables
- Solve problems that require setting up linear equations with two variables
- Perform linear Cost-Volume-Profit and break-even analysis employing:
- The contribution margin approach
- The algebraic approach of solving the cost and revenue functions

SOLVING LINEAR EQUATIONS - AGAIN

Let us look at the example of two linear equations we did in handout 18:

$2x - 3y = -6$ (1)

$x + y = 2$ (2)

Solve for y

We solved for y.

Result:

$y = 2$

SOLVING LINEAR EQUATIONS - PART 2

Let us look at the same example again.

Solve for x

We solved for x.

x = 0

Check your answer
By substituting the values into *each of the equations*
Equation 1:
2x – 3y = -6
x = 0 y = 2
LHS = *2x – 3y = 2(0)-3(2)*
= -*6* **= RHS**
Equation 2 :
x + y = 2
LHS = = x + y = 0 +2 = 2 = RHS
The right side is equal to left hand side. Hence the answer is correct.

LECTURE 20

PERFORM BREAK-EVEN ANALYSIS
EXCEL FUNCTIONS FOR FINANCIAL ANALYSIS

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 18
- MS EXCEL Financial Functions
- Perform Break-Even Analysis.

SETTING UP LINEAR EQUATIONS

Zain purchases the same amount of commodity 1 and 2 each week.
After price increases from Rs. 1.10 to Rs. 1.15 per item of commodity 1 , and from Rs. 0.98 to Rs. 1.14 per item of commodity 2, the weekly bill rose from Rs. 84.40 to Rs. 91.70.
How many items of commodity 1 and 2 are purchased each week?

Setting up Linear Equations

Let x = # of commodity 1
Let y = # of commodity 2

Setting up Linear Equations

Equation 1

1.10x +0.98y= 84.40 (1)
Eliminate x in (1) by Dividing both sides by 1.10.
(1.10x + 0.98y)/1.10 = 84.40/1.10
x + 0.8909y = 76.73

Equation 2

1.15x+1.14y=91.7 (2)
Eliminate x in (2) by Dividing both sides by 1.15
(1.15x + 1.14y)/1.15 = 91.70/1.15
x + 0.9913y = 79.74

Result 1:

x + 0.8909y = 76.73 (3)
x + 0.9913y = 79.74 (4)
Next:
Subtract (4) from (3):

Result 2:

0.1004y = 3.01
y = 3.01/0.1004
Or
y=29.98 (i.e. 30 nos.)
1.10x + 0.98y = 84.40

Substitution

Substitute value of y in (1).
Result:
1.10x + 0.98(29.98) = 84.40
Solve:
1.10x + 29.38 = 84.40
1.10x = 84.40 - 29.38
1.10x = 55.02

Result:

x = 50.02 (i.e. 50 nos.)

New weekly cost

Commodity 1:

50 x 1.15 = 57.50

Commodity 2:

30 x 1.14 = 34.20
Total cost = 91.70

TERMINOLOGY
There are either Business Costs or Expenses.

Fixed Costs
Fixed Costs are such costs that do not change if sales increase or decrease
e.g. rent, property taxes, some forms of depreciation.

Variable Costs
Variable costs do change in direct proportion to sales volume e.g. material costs and direct labour costs.

Break Even Point
Break Even point we discussed earlier. It is a point at which neither a profit nor loss is made.

Contribution Margin
Contribution Margin is the Rs. amount that is found by deducting **ALL Variable Costs** from **Net Sales** and 'contributes' to meeting **Fixed Costs** and making a **'Net Profit'**

Contribution Rate
Contribution Rate is the Rs. amount expressed as a percent (%) of Net Sales.

A CONTRIBUTION MARGIN STATEMENT

	Rs.	%
Net Sales (Price * # Units Sold)	x	100
Less: Variable Costs	x	x
Contribution Margin	x	x
Less: Fixed Costs	x	x
Net Income	x	x

The net sales are calculated by multiplying price per unit with number of units. This figure is treated as 100%. Next, variable costs are specified and deducted from the Net sales to obtain the Contribution Margin. Next, Fixed costs are deducted from the contribution Margin. The result is Net Income. Under the % column, percentage of each item is calculated with respect to the Net Sales.

SCENARIO 1
Market research for a new product indicates that the product can be sold at Rs. 50 per unit. Cost analysis provides the following information:
Fixed Costs (FC) **per period** = Rs. 8640
Variable Costs (VC) = Rs. 30 **per unit.**

Production Capacity **per period** = 900 units
How much does the sale (S) of an additional unit of a firm's product contribute towards increasing its net income?

Formula
Contribution Margin = CM = S – VC
Contribution Rate = CR = CM/S * 100%
***Break Even Point (BEP):**
...in Units (x): Rs. x = (FC / CM)* S
...in Sales Rs. : Rs. x = (FC / CM)* S
...in % of Capacity : BEPin Units/PC*100
* **At Break Even**, Net Profit or Loss = 0
Scenario 1 Summary
The new product can be sold at Rs. 50 per unit. Costs are as follows:
Fixed Costs are Rs. 8640 for the period.
Variable Costs are Rs. 30 per unit
Production Capacity is 900 units per period.

LECTURE 21

PERFORM LINEAR COST-VOLUME PROFIT AND BREAK-EVEN ANALYSIS USING THE CONTRIBUTION MARGIN APPROACH

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 18
- Perform Break-Even Analysis
- MS EXCEL Financial Functions
- .

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 20
- Perform linear cost-volume profit and break-even analysis.
- Using the contribution margin approach

SCENARIO 1

CM = S – VC = 50 - 30 = 20 Rs.
CR = CM/S * 100% = Rs. 20/50 * 100 =40%

Break Even Point:

Units x = FC / CM = 8640/20 = 432 Units
In Rs. x = (FC / CM)* S :
(Rs. 8640/Rs.20)* Rs.50 = Rs.21,600
BEPin units/ PC*100
= 432/ 900*100
= 48% of apacity

SCENARIO 2

The Lighting Division of A Lighting Fitting Manufacturer plans to introduce a new street light based on the following accounting information:
FC = Rs. 3136
VC = Rs.157
S= Rs.185
Capacity = 320 units
Calculate the break even point (BEP)
…in units
…in rupees
…as a percent of capacity

Break Even Point
…in units
= FC / CM
S – VC = CM

= Rs.185 – 157 = Rs.28
= Rs.3136/28 = 112 Units

Break Even Point
…in Rupees
= (FC / CM)* S
= (3136/28) * 185 = 20720 Rs.

Break Even Point
…as a percent of capacity

= BEP(in units)/PC*100

= 112/320 * 100
= 35% of Capacity

SCENARIO 2-1
FC = Rs.3136
VC = Rs.157
S= Rs.185
Capacity = 320 units
Determine the BEP as a % of capacity if FC are reduced to Rs.2688.
Formula: = BEP(in units)/PC*100
Step 1… Find CM
Step 2… Find BEP in units
Step 3… Find % of Capacity
Step 1… Find CM
S = 185
VC = - 157
CM Rs. 28
Step 2… Find BEP in units
= FC/CM
= Rs. 2688/ Rs.28
= 96 Units
Step 3… Find % of Capacity
=BEPin units /PC*100
= 96/320*100
= 30% of Capacity
SCENARIO 2-2

FC = Rs.3136 VC = Rs.157
S = Rs.185
Capacity = 320 units
VC = S*80% = Rs.148
Determine the BEP as a % of capacity if FC are increased to Rs.4588, and VC reduced to 80% of S.
= BEP(in units)/PC*100

Step 1… Find CM

S = 185
VC = -148
CM = Rs. 37

Step 2… Find BEP in units

= FC/CM
= Rs. 4588 / Rs. 37
= 124 Units

Step 3… Find % of Capacity

=BEPin units /PC*100
= 124/320*100
= 39% of Capacity

SCENARIO 2 -3

FC = Rs. 3136
VC = Rs.157
S= Rs.185
Capacity = 320 units
Determine the BEP as a % of capacity if S is reduced to Rs.171.
= BEP(in units)/PC*100

Step 1… Find CM

S = 171
VC = -157
CM = Rs. 14

Step 2… Find BEP in units

= FC/CM
= Rs. 3136/ Rs. 14
= 224 Units

Step 3… Find % of Capacity

=BEPin units /PC*100
= 224/320*100
= 70 % of Capacity

LECTURE 22
PERFORM LINEAR COST-VOLUME PROFIT AND BREAK-EVEN ANALYSIS

OBJECTIVES

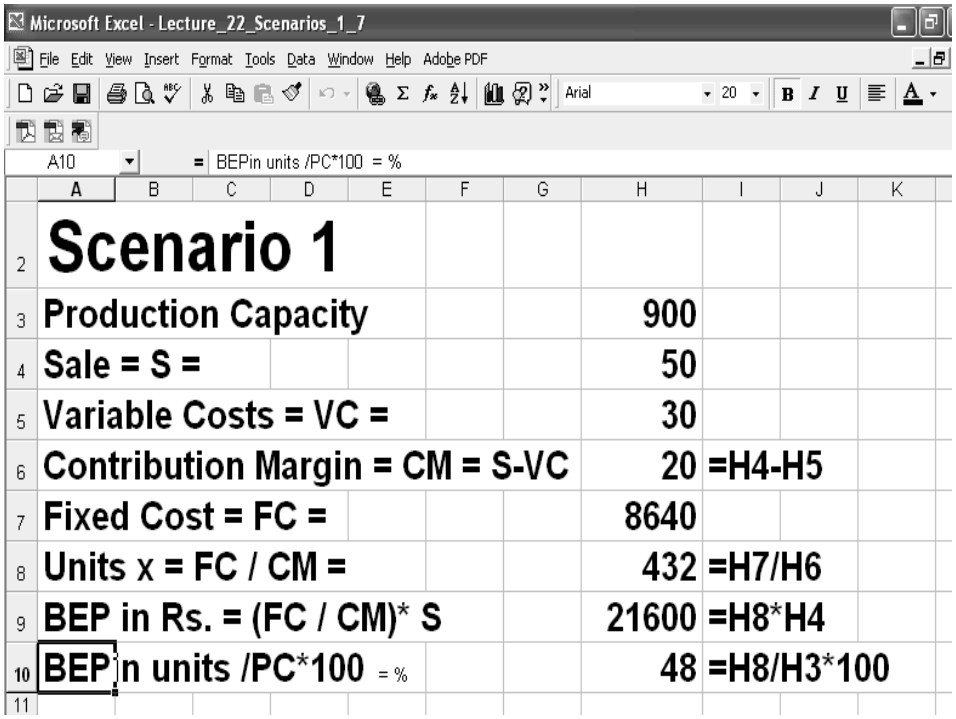
The objectives of the lecture are to learn about:

- Review Lecture 21
- Perform Linear Cost-Volume Profit and Break-Even analysis.
- Using Microsoft Excel

SCENARIO 1

Let us look at different scenarios for calculation of contribution margin and net profit. The explanations are given in the slides.

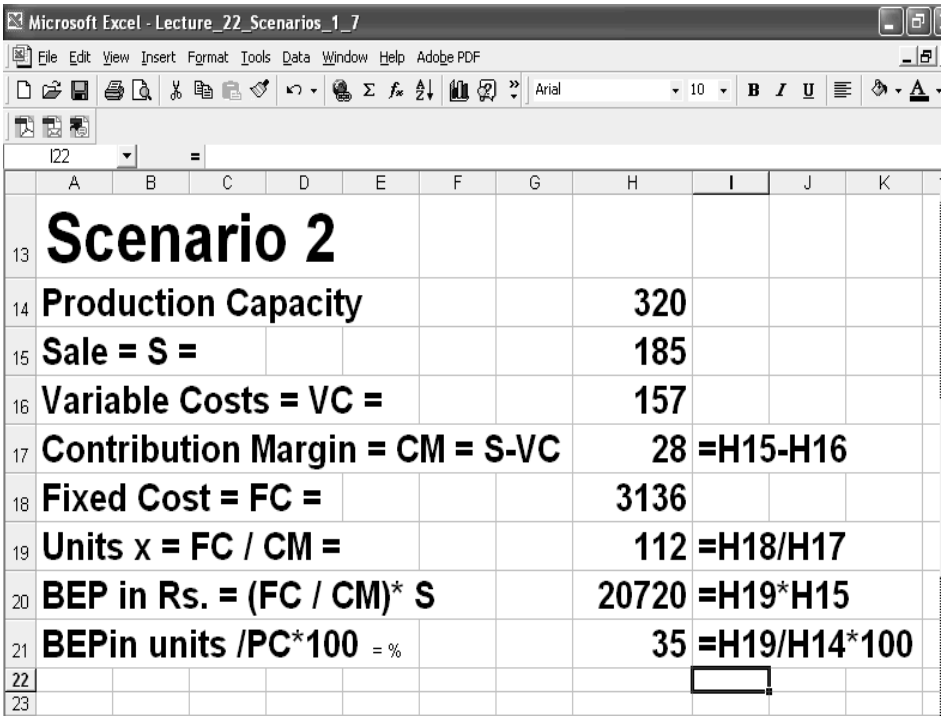
The Break Even in Rs. Is 21,600. The break Even in units is 48.



	A	H	I
2	Scenario 1		
3	Production Capacity	900	
4	Sale = S =	50	
5	Variable Costs = VC =	30	
6	Contribution Margin = CM = S-VC	20	=H4-H5
7	Fixed Cost = FC =	8640	
8	Units x = FC / CM =	432	=H7/H6
9	BEP in Rs. = (FC / CM)* S	21600	=H8*H4
10	BEP n units /PC*100 = %	48	=H8/H3*100
11			

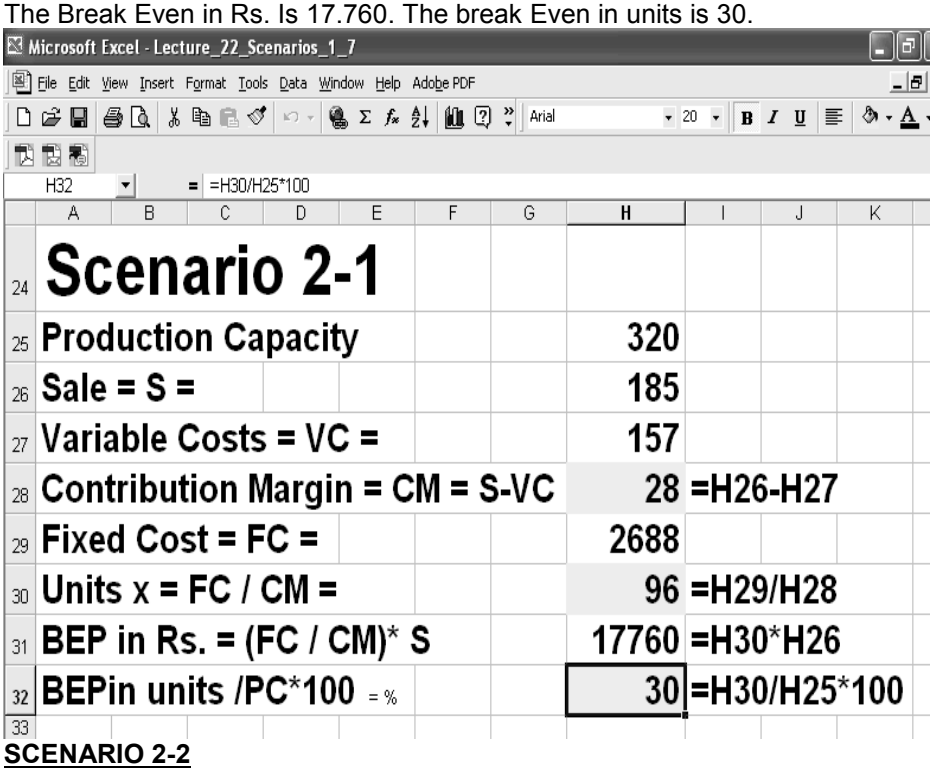
SCENARIO 2

The Break Even in Rs. Is 20,720. The break Even in units is 35.



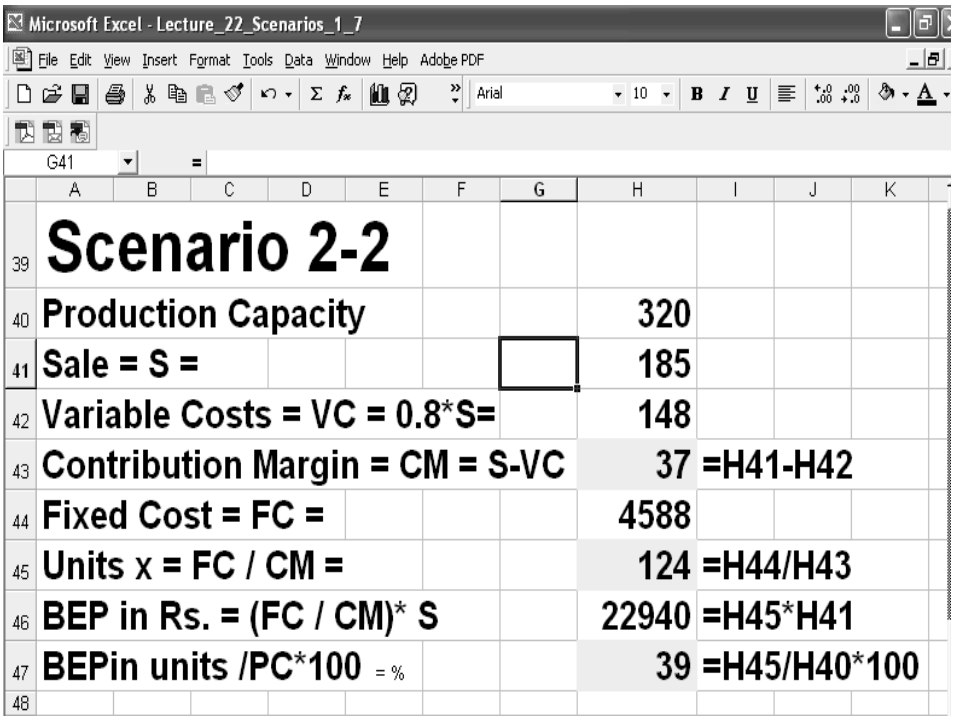
SCENARIO 2-1

The Break Even in Rs. Is 17.760. The break Even in units is 30.



SCENARIO 2-2

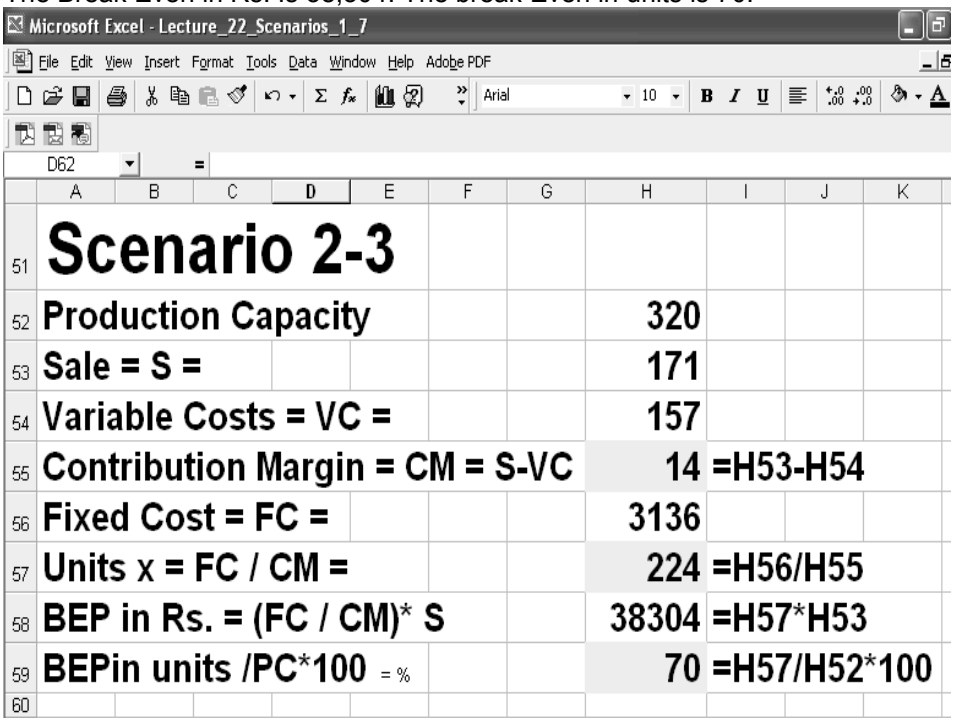
The Break Even in Rs. Is 22,940. The break Even in units is 39.


Microsoft Excel - Lecture_22_Scenarios_1_7

	A	B	C	D	E	F	G	H	I	J	K
39	**Scenario 2-2**										
40	Production Capacity							320			
41	Sale = S =							185			
42	Variable Costs = VC = 0.8*S=							148			
43	Contribution Margin = CM = S-VC							37	=H41-H42		
44	Fixed Cost = FC =							4588			
45	Units x = FC / CM =							124	=H44/H43		
46	BEP in Rs. = (FC / CM)* S							22940	=H45*H41		
47	BEPin units /PC*100 = %							39	=H45/H40*100		
48											

SCENARIO 2-3

The Break Even in Rs. Is 38,304. The break Even in units is 70.


Microsoft Excel - Lecture_22_Scenarios_1_7

	A	B	C	D	E	F	G	H	I	J	K
51	**Scenario 2-3**										
52	Production Capacity							320			
53	Sale = S =							171			
54	Variable Costs = VC =							157			
55	Contribution Margin = CM = S-VC							14	=H53-H54		
56	Fixed Cost = FC =							3136			
57	Units x = FC / CM =							224	=H56/H55		
58	BEP in Rs. = (FC / CM)* S							38304	=H57*H53		
59	BEPin units /PC*100 = %							70	=H57/H52*100		
60											

SCENARIO 2-4

FC = Rs. 3136 VC = Rs. 157 S= Rs. 185 Capacity = 320units

Determine the NI if 134 units are sold!

Formula for Net Income

NI = #Units above BEP*CM

= BEP (in units)/PC*100

Step 1… Find CM

S = 185

VC = -157

CM = Rs. 28

(CM of Rs.28 per unit)

Step 2… Find BEP in units

= FC/CM

= Rs. 3136/ Rs. 28

= 112 Units

Step 3… Find units over BEP Units

Sold 134

BEP 112

Over BEP 22

Hence:

Company had a NI of 22 * Rs. 28 = Rs. 616

Scenario 2-5

FC = Rs. 3136

VC = Rs.157

S= Rs.185

Capacity = 320 units

What unit sales will generate NI of Rs. 2000?

Formula for Net Income

#Units above BEP = NI/CM

Step 1… Find CM

S = 185

VC = -157

CM = Rs. 28

(CM of Rs.28 per unit)

Step 2… Find BEP in units

= FC/CM

= Rs. 3136/ Rs. 28

= 112 Units

Step 3… Find units over BEP

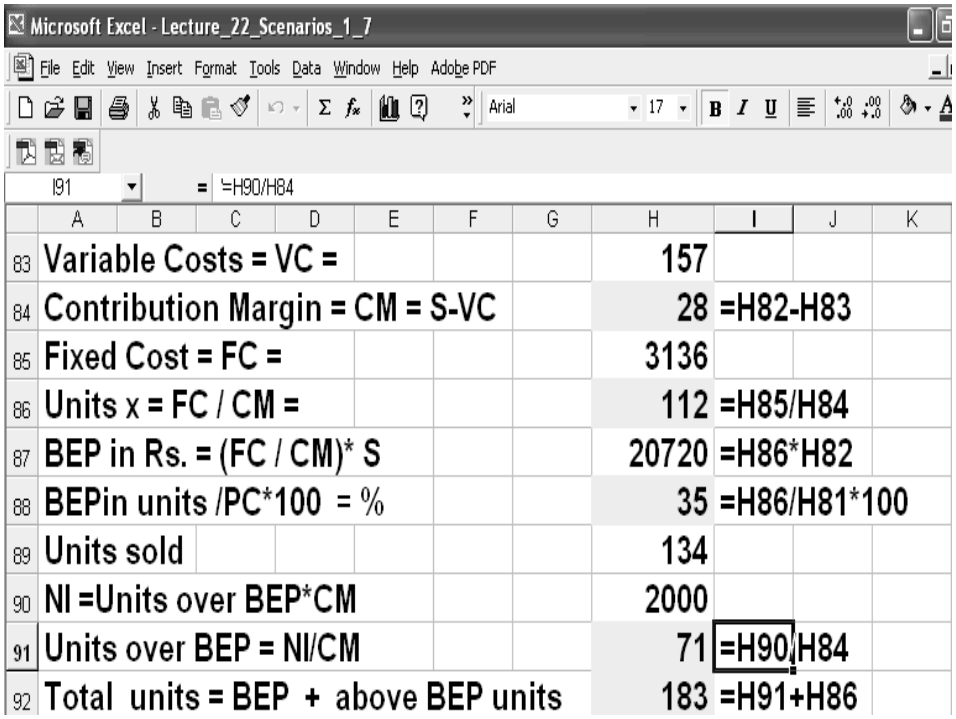
NI/CM = Rs. 2000/Rs. 28 per Unit

= 72 Units above Break Even

Hence:

72 Units above BEP + 112 BEP Units

= Total Sales Units = 184



Scenario 2-6

FC = Rs. 3136
VC = Rs.157
S= Rs.185
Capacity = 320 units
What are the unit sales if there is a Net Loss of Rs.336?

Formula

Units below BEP = (NI)/CM

Step 1… Find CM

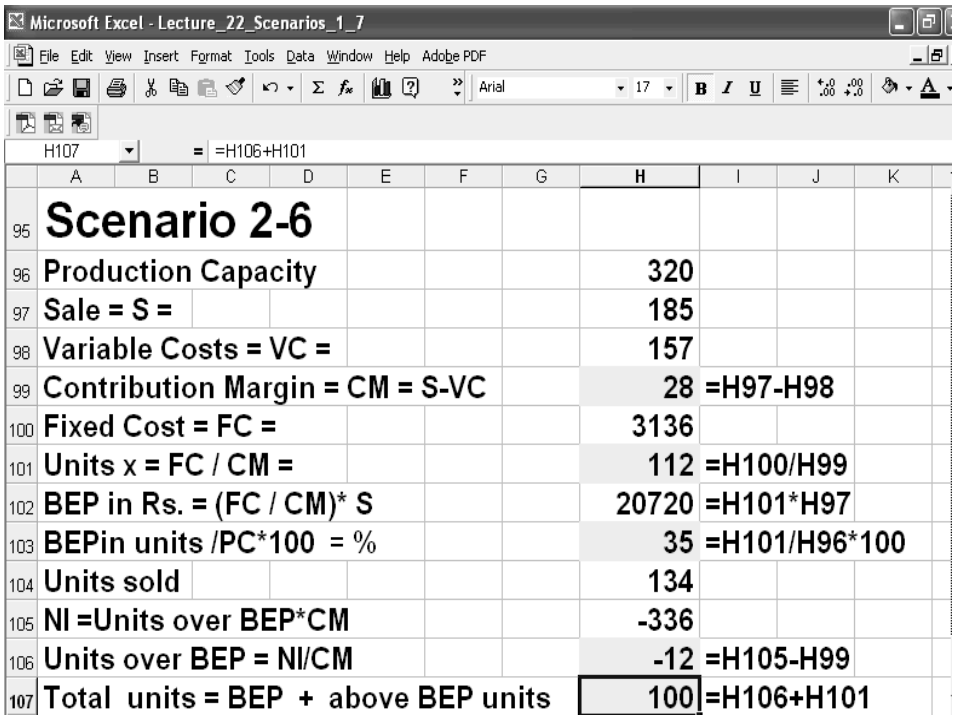
S = 185
VC = -157
CM = Rs. 28
(CM of Rs.28 per unit)

Step 2… Find BEP in units

= FC/CM
= Rs. 3136/ Rs. 28
= 112 Units

Step 3… Find units below BEP NI/CM

= Rs. 336/Rs. 28 per Unit
= 12 Units below Break Even
Hence:
112 BEP - 12 Units Below
= Total Sales Units = 100



	A	B	C	D	E	F	G	H	I	J	K
95	**Scenario 2-6**										
96	Production Capacity							320			
97	Sale = S =							185			
98	Variable Costs = VC =							157			
99	Contribution Margin = CM = S-VC							28	=H97-H98		
100	Fixed Cost = FC =							3136			
101	Units x = FC / CM =							112	=H100/H99		
102	BEP in Rs. = (FC / CM)* S							20720	=H101*H97		
103	BEPin units /PC*100 = %							35	=H101/H96*100		
104	Units sold							134			
105	NI =Units over BEP*CM							-336			
106	Units over BEP = NI/CM							-12	=H105-H99		
107	Total units = BEP + above BEP units							100	=H106+H101		

FC = Rs. 3136
VC = Rs.157
S= Rs.185
Capacity = 320 units
The company operates at 85% capacity.
Find the Profit or Loss.

Formula

units above BEP *CM = N

Step 1… Find CM

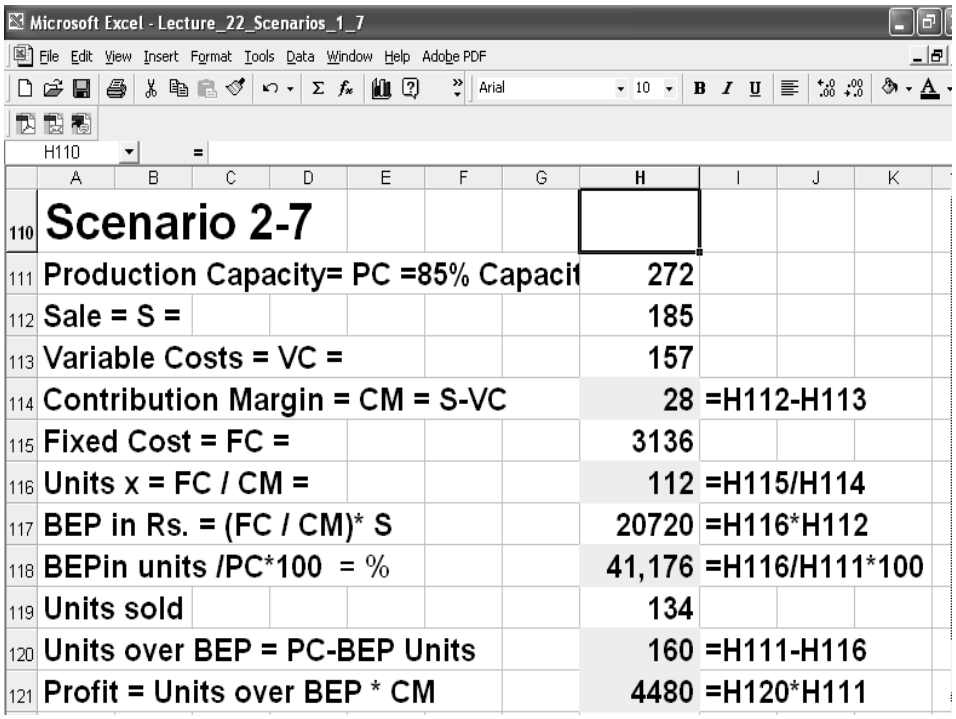
S = 185
VC = -157
CM = Rs. 28
(CM of Rs.28 per unit)

Step 2… Find BEP in units

= FC/CM
= Rs. 3136/ Rs. 28
= 112 Units

Step 3… Find units over BEP

320*.85 = 2
Units
Production 272
BEP 112
Over BEP 160
Hence:
160 Units * 28 = Profit 4480 Rs.



	A	B	C	D	E	F	G	H	I	J	K
110	**Scenario 2-7**										
111	Production Capacity= PC =85% Capacit							272			
112	Sale = S =							185			
113	Variable Costs = VC =							157			
114	Contribution Margin = CM = S-VC							28	=H112-H113		
115	Fixed Cost = FC =							3136			
116	Units x = FC / CM =							112	=H115/H114		
117	BEP in Rs. = (FC / CM)* S							20720	=H116*H112		
118	BEPin units /PC*100 = %							41,176	=H116/H111*100		
119	Units sold							134			
120	Units over BEP = PC-BEP Units							160	=H111-H116		
121	Profit = Units over BEP * CM							4480	=H120*H111		

CASE

Company A's year end operating results were as follows:
Total Sales of Rs. 375,000
Operated at 75% of capacity
Total Variable Costs were Rs. 150,000
Total Fixed Costs were Rs. 180,000
What was Company A's BEP expressed in rupees of sales?

LECTURE 23

STATISTICAL DATA REPRESENTATION

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 22
- Statistical Data Representation.

MODULE 5

Statistical data representation
(Lecture 23)
Measures of central tendency
(Lectures 24-25)
Measures of dispersion and skewness
(Lectures 26-27)

MODULE 6

Correlation
(Lecture 28-29)
Line Fitting
(Lectures 30-31)
Time Series and Exponential Smoothing
(Lectures 32-33)

MODULE 7

Factorials
Permutations and Combinations
(Lecture 34)
Elementary Probability
(Lectures 35-36)
Chi-Square
(Lectures 37)
Binomial Distribution
(Lectures 38)

MODULE 8

Patterns of probability: Binomial, Poisson and Normal Distributions
(Lecture 39-41)
Estimating from Samples: Inference
(Lectures 42-43)
Hypothesis testing: Chi-Square Distribution
(Lectures 44-45)
End-Term Examination

STATISTICAL DATA

Information is collected by government departments, market researchers, opinion pollsters and others.
Information then has to be organised and presented in a way that is easy to understand

BASIS FOR CLASSIFICATION

1. Qualitative: Attributes: sex, religion
2. QuantitativeCharacteristics: Heights, weights, incomes etc.
3. Geographical: Regions: Provinces, divisions etc.
4. Chronological or Temporal
5. By time of occurrence: Time series

TYPES OF CLASSIFICATION

There are different types of classifications.

- One-way
  One characteristic: Population
- Two-way
  Two characteristics at a time
- Three-way
  Three characteristics at a time

METHODS OF PRESENTATION

Different methods of representation are:

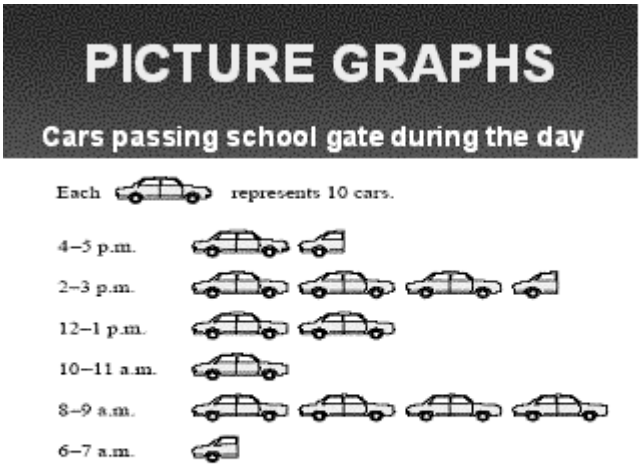
- Text
  "The majority of population of Punjab is located in rural areas."
- Semitabular
  Data in rows
- Tabular
  Tables with rows and columns
- Graphic
- Charts and graphs

TYPES OF GRAPHS

- Column Graphs
- Line Graphs
- Circle Graphs (Sector Graphs)
- Conversion Graphs
- Travel Graphs
- Statistical Graphs
- Frequency Tables
- Histograms
- Frequency distributions
- Cumulative Distributions

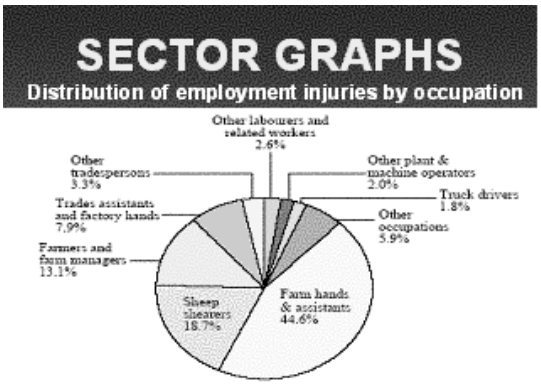
PICTURE GRAPHS

Picture graphs use the picture as one unit. In the example below, one car represents 10 cars.



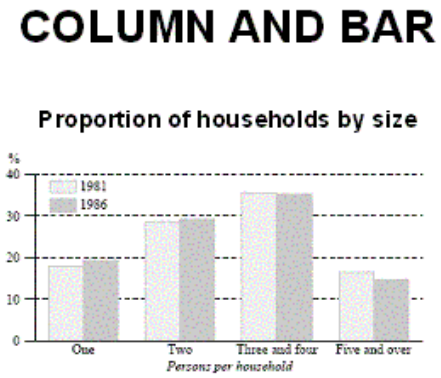
SECTOR GRAPHS

Sector graphs use the division of a circle into different sectors. The full circle is 360 degrees. For each percentage, degees are calculated and sectors plotted.



COLUMN AND BAR GRAPHS

The following slide gives the Proportion of households by size in the form of a Column and Bar graph.



LINE GRAPHS

Line graphs are the most commonly used graphs. Here the data of one variable (say Height) is plotted against data of the other variable (say Age).